Finding Relationships Among Variables

BUS 735: Business Decision Making and Research

Goals

- Specific goals:
  - Detect *relationships* between variables.
  - Be able to prescribe appropriate statistical methods for measuring relationship based on scale of measurement.
  - Detect how outcome variables can be explained by one or more explanatory variables.
- Learning objectives:
  - LO1: Construct and test hypotheses using a variety of bivariate statistical methods to compare characteristics between two populations.
  - LO2: Construct and use advanced multivariate models to identify complex relationships among multiple variables; including regression models, limited dependent variable models, and analysis of variance and covariance models.

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Correlation

- A **correlation** exists between two variables when one of them is related to the other in some way.
- **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.
  - Parametric test for interval or ratio data
  - Null hypothesis: there is zero linear correlation between two variables.
  - Alternative hypothesis: there is a linear correlation (either positive or negative) between two variables.
  - Measures strength of *linear* relationship
- **Spearman linear correlation coefficient**
  - Non-parametric test for ordinal, interval, and ratio data
  - Pearson computation with *ranks* instead of actual data
  - Same hypotheses.
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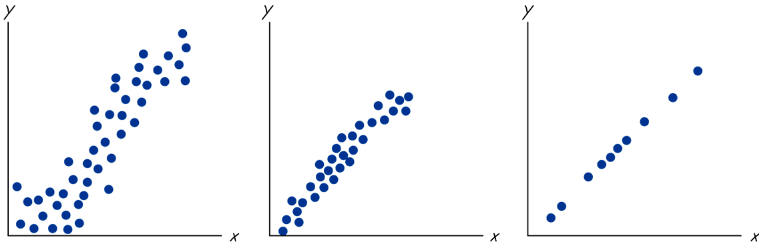
Positive linear correlation



(a) Positive correlation between x and y

(b) Strong positive correlation between x and y

(c) Perfect positive correlation between x and y

- Positive correlation: two variables move in the same direction.
- Stronger correlation: closer correlation is to 1.0
- Perfect positive correlation: $\rho = 1.0$

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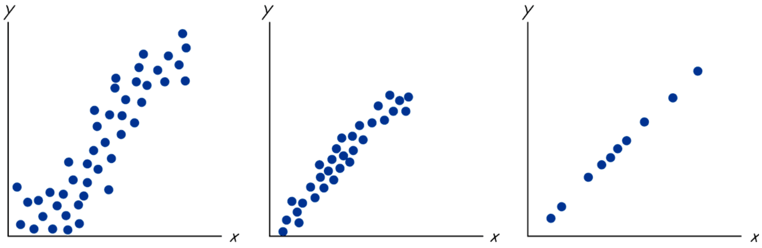

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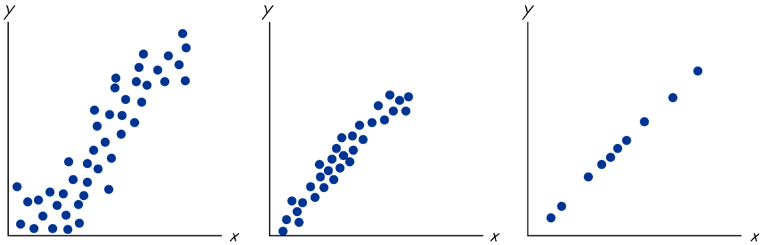

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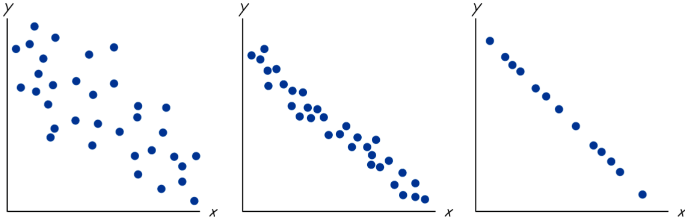

(d) Negative correlation between x and y

(e) Strong negative correlation between x and y

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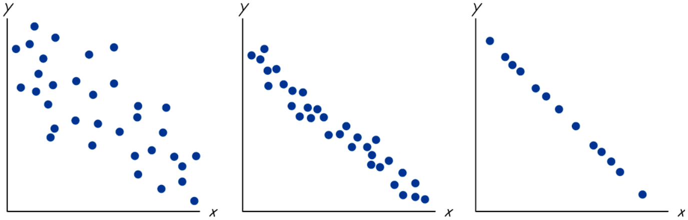

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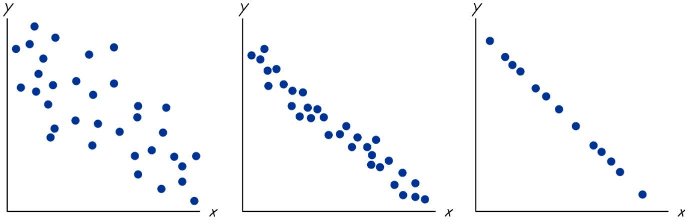

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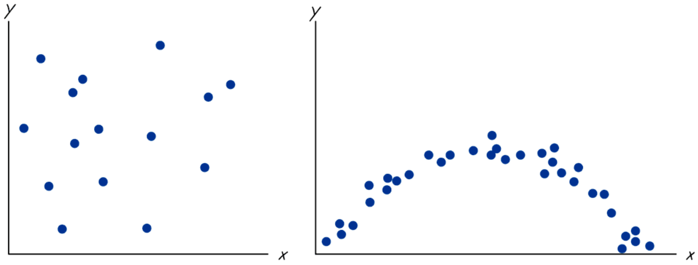

(g) No correlation between x and y

(h) Nonlinear relationship between x and y

- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
  - Cannot use regular correlation to detect this.

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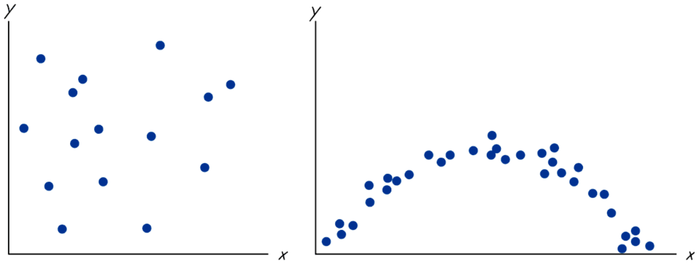

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Chi-Square Test for Independence

- Used to determine if two categorical variables (eg: nominal) are related.
- Example: Suppose a hotel manager surveys guest who indicate they will not return:

	Reason for Not Returning		
Reason for Stay	Price	Location	Amenities
Personal/Vacation	56	49	0
Business	20	47	27

- Data in the table are always frequencies that fall into individual categories.
- Could use this table to test if two variables are independent.

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Bivariate Regression

- Regression line: equation of the line that describes the linear relationship between variable x and variable y.
- Need to assume that *independent variables* influence *dependent variables*.
  - x: *independent* or *explanatory* variable.
  - y: *dependent* or *outcome* variable.
  - Variable x can influence variable y, but not vice versa.
- Example: How does advertising expenditures affect sales revenue?

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Regression line

Population regression line:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

- The population coefficients β_0 and β_1 describing the relationship between x and y are unknown.
- Since x and y are not perfectly correlated, ϵ_i is the error term.

Sample regression line:

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- Not perfectly correlated, e_i is the sample error term.

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Predicted values and residuals

For a given x_i, the **predicted value** for y_i, denoted $\hat{y}_i$, is...

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- This is not likely be the actual value for y_i.

Residual is the difference *in the sample* between the actual value of y_i and the predicted value, $\hat{y}$.

$$e_i = y_i - \hat{y}_i = y_i - b_0 - b_1 x_i$$

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Multiple Regression

Multiple regression line **(population)**:

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Multiple regression line **(sample)**:

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Interpreting the slope

- Interpreting the slope, β: amount the y is predicted to increase when increasing x by one unit.
- When $\beta < 0$ there is a negative linear relationship.
- When $\beta > 0$ there is a positive linear relationship.
- When $\beta = 0$ there is no linear relationship between x and y.
- Statistical packages report sample estimates for coefficients, along with...
  - Standard errors of the coefficients
  - T-test statistics for $H_0 : \beta = 0$.
  - P-values of the T-tests.
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Adjusted R^2

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- Adding such variables increases in-sample fit, but will likely hurt out-of-sample forecasting accuracy.
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- Very, very, very similar to ANOVA F-test.
- $H_0 : \beta_1 = \beta_2 = ... = \beta_k = 0$.
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- Using the normal distribution to compute p-values depends on results from the Central Limit Theorem.
- Sufficiently large sample size (much more than 30).
  - Useful for normality result from the Central Limit Theorem
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