Finding Relationships Among Variables

BUS 230: Business and Economic Research and Communication

1

Goals

- Specific goals:
  - Re-familiarize ourselves with basic statistics ideas: sampling distributions, hypothesis tests, p-values.
  - Be able to distinguish different types of data and prescribe appropriate statistical methods.
  - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
  - LO2: Interpret data using statistical analysis.
  - LO2.3: Formulate conclusions and recommendations based upon statistical results.

What to Look For

- There is a closed-book, closed-note quiz tomorrow.
- For each test, remember the following:
  - In plain English, be able to describe the purpose of the test.
  - Know whether the test is a parametric test or a non-parametric test.
  - Know the null and alternative hypotheses.
  - Know what types of variables are appropriate for applying the test.

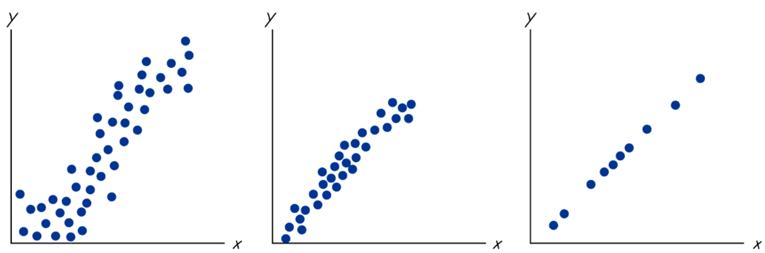
2 Relationships Between Two Variables

2.1 Correlation

Correlation

- A **correlation** exists between two variables when one of them is related to the other in some way.
- The **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.
  - Parametric test!
  - Null hypothesis: there is zero linear correlation between two variables.
  - Alternative hypothesis: there is a linear correlation (either positive or negative) between two variables.
- Spearman's Rank Test
  - Non-parametric test.
  - Behind the scenes - replaces actual data with their *rank*, computes the Pearson using ranks.
  - Same hypotheses.

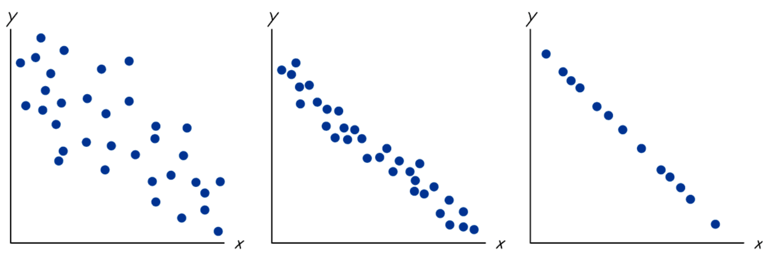
Positive linear correlation



(a) Positive correlation between x and y **(b)** Strong positive correlation between x and y **(c)** Perfect positive correlation between x and y

- Positive correlation: two variables move in the same direction.
- Stronger the correlation: closer the correlation coefficient is to 1.
- Perfect positive correlation: $\rho = 1$

Negative linear correlation



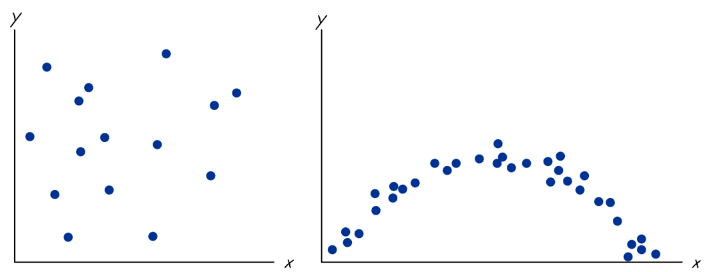
(d) Negative correlation between x and y

(e) Strong negative correlation between x and y

(f) Perfect negative correlation between x and y

- Negative correlation: two variables move in opposite directions.
- Stronger the correlation: closer the correlation coefficient is to -1.
- Perfect negative correlation: $\rho = -1$

No linear correlation



(g) No correlation between x and y

(h) Nonlinear relationship between x and y

- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
  - Cannot use regular correlation to detect this.

2.2 Chi-Squared Test of Independence

Chi-Squared Test for Independence

- Used to determine if two categorical variables (eg: nominal) are related.
- Example: Suppose a hotel manager surveys guest who indicate they will not return:

	Reason for Not Returning		
Reason for Stay	Price	Location	Amenities
Personal/Vacation	56	49	0
Business	20	47	27

- Data in the table are always frequencies that fall into individual categories.
- Could use this table to test if two variables are independent.

Test of independence

- **Null hypothesis**: there is no relationship between the row variable and the column variable (independent)
- **Alternative hypothesis**: There is a relationship between the row variable and the column variable (dependent).
- Test statistic:
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$
- O: observed frequency in a cell from the contingency table.
- E: expected frequency computed with the *assumption that the variables are independent*.
- Large χ^2 values indicate variables are dependent (reject the null hypothesis).

3 Regression

3.1 Single Variable Regression

Regression

- Regression line: equation of the line that describes the linear relationship between variable x and variable y.
- Need to assume that *independent variables* influence *dependent variables*.
  - x: *independent* or *explanatory* variable.
  - y: *dependent* variable.
  - Variable x can influence the value for variable y, but not vice versa.
- Example: How does advertising expenditures affect sales revenue?

Regression line

- Population regression line:
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$
- The actual coefficients β_0 and β_1 describing the relationship between x and y are unknown.

- Use sample data to come up with an estimate of the regression line:
$$y_i = b_0 + b_1 x_i + e_i$$
- Since x and y are not perfectly correlated, still need to have an error term.

Predicted values and residuals

- Given a value for x_i, can come up with a **predicted value** for y_i, denoted $\hat{y}_i$.
$$\hat{y}_i = b_0 + b_1 x_i$$
- This is not likely be the actual value for y_i.
- **Residual** is the difference *in the sample* between the actual value of y_i and the predicted value, $\hat{y}$.
$$e_i = y_i - \hat{y} = y_i - b_0 - b_1 x_i$$

3.2 Multiple Regression

Multiple Regression

- Multiple regression line (population):
$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_2 + ... + \beta_{k-1} x_{k-1} + \epsilon_i$$
- Multiple regression line (sample):
$$y_i = b_0 + b_1 x_{1,i} + b_2 x_2 + ... + b_k x_k + e_i$$
  - k: number of parameters (coefficients) you are estimating.
  - ϵ_i: error term, since linear relationship between the x variables and y are not perfect.
  - e_i: residual = the difference between the predicted value $\hat{y}$ and the actual value y_i.

Interpreting the slope

- Interpreting the slope, β: amount the y is predicted to increase when increasing x by one unit.
- When $\beta < 0$ there is a negative linear relationship.
- When $\beta > 0$ there is a positive linear relationship.
- When $\beta = 0$ there is no linear relationship between x and y.
- SPSS reports sample estimates for coefficients, along with...
  - Estimates of the standard errors.
  - T-test statistics for $H_0 : \beta = 0$.
  - P-values of the T-tests.
  - Confidence intervals for the coefficients.

3.3 Variance Decomposition

Sum of Squares Measures of Variation

- **Sum of Squares Regression (SSR)**: measure of the amount of variability in the dependent (Y) variable that is explained by the independent variables (X's).
$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$
- **Sum of Squares Error (SSE)**: measure of the unexplained variability in the dependent variable.
$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Sum of Squares Measures of Variation

- **Sum of Squares Total (SST)**: measure of the total variability in the dependent variable.
$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$
- SST = SSR + SSE.

Coefficient of determination

- The **coefficient of determination** is the percentage of variability in y that is explained by x.
$$R^2 = \frac{SSR}{SST}$$
- R^2 will always be between 0 and 1. The closer R^2 is to 1, the better x is able to explain y.
- The more variables you add to the regression, the higher R^2 will be.

Adjusted R^2

- R^2 will likely increase (slightly) even by adding nonsense variables.
- Adding such variables increases in-sample fit, but will likely hurt out-of-sample forecasting accuracy.
- The Adjusted R^2 penalizes R^2 for additional variables.

$$R^2_{\text{adj}} = 1 - \frac{n-1}{n-k-1}\left(1 - R^2\right)$$

- When the adjusted R^2 increases when adding a variable, then the additional variable really did help explain the dependent variable.
- When the adjusted R^2 decreases when adding a variable, then the additional variable does not help explain the dependent variable.

F-test for Regression Fit

- F-test for Regression Fit: Tests if the regression line explains the data.
- Very, very, very similar to ANOVA F-test.
- $H_0 : \beta_1 = \beta_2 = ... = \beta_k = 0$.
- H_1 : At least one of the variables has explanatory power (i.e. at least one coefficient is not equal to zero).

$$F = \frac{SSR/(k-1)}{SSE/(n-k)}$$

- Where k is the number of explanatory variables.

3.4 Regression Assumptions

Assumptions from the CLT

- Using the normal distribution to compute p-values depends on results from the Central Limit Theorem.
- Sufficiently large sample size (much more than 30).
  - Useful for normality result from the Central Limit Theorem
  - Also necessary as you increase the number of explanatory variables.
- Normally distributed dependent and independent variables
  - Useful for small sample sizes, but not essential as sample size increases.
- Types of data:
  - Dependent variable must be interval or ratio.
  - Independent variable can be interval, ratio, *or a dummy variable.*

Crucial Assumptions for Regression

- Linearity: a straight line reasonably describes the data.
  - Exceptions: experience on productivity, ordinal data like education level on income.
  - Consider transforming variables.
- Stationarity:
  - The central limit theorem: behavior of statistics as sample size approaches infinity!
  - The mean and variance must exist and be constant.
  - Big issue in economic and financial time series.
- Exogeneity of explanatory variables.
  - Dependent variable must not influence explanatory variables.
  - Explanatory variables must not be influenced by excluded variables that can influence dependent variable.
  - Example problem: how does advertising affect sales?