Statistical Significance and Univariate and Bivariate Tests

BUS 230: Business and Economics Research and Communication

Goals

- Specific goals:
  - Re-familiarize ourselves with basic statistics ideas: sampling distributions, hypothesis tests, p-values.
  - Be able to distinguish different types of data and prescribe appropriate statistical methods.
  - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
  - LO2: Interpret data using statistical analysis.
  - LO2.3: Formulate conclusions and recommendations based upon statistical results.

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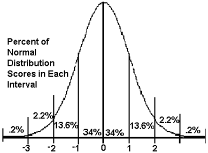
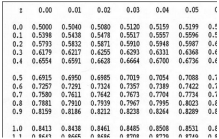
Probability Distribution

- **Probability distribution:** summary of all possible values a variable can take along with the probabilities in which they occur.
- Usually displayed as:

Picture



Table



z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531

Formula

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

- **Normal distribution:** often used "bell shaped curve", reveals probabilities based on how many standard deviations away an event is from the mean.

Sampling distribution

- Imagine taking a sample of size 100 from a population and computing some kind of statistic.
- The statistic you compute can be anything, such as: mean, median, proportion, difference between two sample means, standard deviation, variance, or anything else you might imagine.
- Suppose you repeated this experiment over and over: take a sample of 100 and compute and record the statistic.
- A **sampling distribution** is the probability distribution *of the statistic*
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  NO! They may coincidentally have the same shape though.

Example

- Sampling Distribution Simulator
- In reality, you only do an experiment once, so the sampling distribution is a hypothetical distribution.
- Why are we interested in this?

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- The average of the sample statistics is equal to the true population parameter.
- Want the variance *of the sampling distribution* to be as small as possible. Why?
- Want the *sampling distribution* to be normal, regardless of the distribution of the population.

Central Limit Theorem

- Given:
  - Suppose a RV x has a distribution (it need not be normal) with mean μ and standard deviation σ.
  - Suppose a *sample mean* ($\bar{x}$) is computed from a sample of size n.
- Then, if n is sufficiently large, the sampling distribution of $\bar{x}$ will have the following properties:
  - The sampling distribution of $\bar{x}$ will be normal.
  - The mean of the sampling distribution will equal the mean of the population (consistent):

$$\mu_{\bar{x}} = \mu$$

  - The standard deviation of the sampling distribution will decrease with larger sample sizes, and is given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

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Example 2

Suppose average birth weight is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.

What is the probability that a randomly selected baby will have a weight of 7.5lbs or more? What do you need to assume to answer this question?

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Example 3

- Suppose average birth weight of all babies is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.
- Suppose you collect a sample of 30 newborn babies whose mothers smoked during pregnancy.
- Suppose you obtained a sample mean $\bar{x} = 5$ lbs. If you assume the mean birth weight of babies whose mothers used illegal drugs has the same sampling distribution as the rest of the population, what is the probability of getting a sample mean this low or lower?

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That is, if using drugs during pregnancy actually does still lead to an average birth weight of 7 pounds, there was only a 0.000131 (or 0.0131%) chance of getting a sample mean as low as six or lower.

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That is, if using drugs during pregnancy actually does still lead to an average birth weight of 7 pounds, there was only a 0.000131 (or 0.0131%) chance of getting a sample mean as low as six or lower. This is an extremely unlikely event if the assumption is true. Therefore it is likely the assumption is not true.

Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
  - Example: The population mean for systolic blood pressure is 120.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example about birth weights with mothers who use drugs.
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Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) *is equal to* some claimed value.
  - H_0: $\mu = 7$.
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter differs from the value given in the null hypothesis.
- Pick *only one* of the following for your alternative hypothesis. Which one depends on your research question.
  - H_a: $\mu < 7$.
  - H_a: $\mu > 7$.
  - H_a: $\mu \neq 7$.
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an "innocent until proven guilty" policy.

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- Interpretation: If the null hypothesis is correct, than the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence of the null hypothesis.
- The p-value is therefore a measure of *statistical significance*.
  - If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
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Types of Data

- Nominal data: consists of categories that cannot be ordered in a meaningful way.
- Ordinal data: order is meaningful, but not the distances between data values.
  - Excellent, Very good, Good, Poor, Very poor.
- Interval data: order is meaningful, *and* distances are meaningful. However, there is *no natural zero*.
  - Examples: temperature, time.
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- Test whether the population mean is equal or different to some value.
- Uses the sample mean its statistic.
- T-test is used instead of Z-test for reasons you may have learned in a statistics class. Interpretation is the same.
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Example Questions

- Suppose we have a dataset of individual schools and the average pay for teachers in each school, and the level of spending as a ratio of the number of students (spending per pupil).
  - Show some descriptive statistics for teacher pay and expenditure per pupil.
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- Example: percentage of consumers of soda who prefer Pepsi over Coke.

$$\text{Sample proportion} = \frac{\text{Number of items that has characteristic}}{\text{sample size}}$$

- Example questions:
  - Are more than 50% of potential voters most likely to vote for Barack Obama in the next presidential election?
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  - Ordinal data: cannot compute sample means (they are meaningless), only median is meaningful.
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Difference in Means (Independent Samples)

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- Independent samples means you have different individuals in your two sample groups.
- Examples:
  - Compare sales volume for stores that advertise versus those that do not.
  - Compare production volume for employees that have completed some type of training versus those who have not.
- Statistic: Difference in the sample means $(\bar{x}_1 - \bar{x}_2)$.
- Hypotheses:
  - Null hypothesis: the difference between the two means is zero.
  - Alternative hypothesis: the difference is [above/below/not equal] to zero.

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Nonparametric Tests for Differences in Medians

- Mann-Whitney U test: nonparametric test to determine difference in *medians*.
- Can you suggest some examples?
- Assumptions:
  - Samples are independent of one another.
  - The underlying distributions have the same shape (i.e. only the location of the distribution is different).
  - It has been argued that violating the second assumption does not severely change the sampling distribution of the Mann-Whitney U test.
- Null hypothesis: medians for the two populations are the same.
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Dependent Samples - Paired Samples

- Use a **paired sampled test** if instead the two samples have the same individuals before and after some treatment.
- Examples:
  - The Biggest Loser: Compare the weight of people on the show before the season begins and one year after the show concludes.
  - Training session: Are workers more productive 6 months after they attended some training session versus before the training session.
- Really simple: for each individual subtract the before treatment measure from the after treatment measure (or vice-versa).
- Treat your new series as a single series.
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Conclusions

- Ideas to keep in mind:
  - What is a sampling distribution? What does it imply about p-values and statistical significance?
  - When it is appropriate to use parametric versus non-parametric methods.
  - Most univariate and bivariate questions have a parametric and non-parametric approach.
- Next class: closed-book, closed note quiz (so study!); and an in-class exercise practicing this stuff in SPSS.
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