# Regression Analysis

BUS 735: Business Decision Making and Research

## Goals of this section



- Specific goals:
  - Learn how detect linear relationships between variables.
  - Learn how to detect relationships between ordinal and categorical variables.
  - Learn how to estimation the relationship between many variables.
- Learning objectives:
  - LO2: Be able to construct and use multiple regression models (including some limited dependent variable models) to construct and test hypotheses considering complex relationships among multiple variables.
  - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.
  - LO7: Have a sound familiarity of various statistical and quantitative methods in order to be able to approach a business decision problem and be able to select appropriate methods to answer the question.

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# Chi-Squared Test for Independence



- Used to determine if two categorical variables are related.
- Example: Suppose a hotel manager surveys guest who indicate they will not return:

| | Reason for Not Returning | | |
|---|---|---|---|
| Reason for Stay | Price | Location | Amenities |
| Personal/Vacation | 37 | 54 | 0 |
| Business | 34 | 55 | 19 |

- Data in the table are always frequencies that fall into individual categories.
- Could use this table to test if two variables are independent.

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## Test of independence



- **Null hypothesis**: there is no relationship between the row variable and the column variable.
- **Alternative hypothesis**: The two variables are dependent.
- Test statistic:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- $O$: observed frequency in a cell from the contingency table.
- $E$: expected frequency assuming variables are independent.
- Large $\chi^2$ values indicate variables are dependent (reject the null hypothesis).

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# Correlation



- A **correlation** exists between two variables when one of them is related to the other in some way.
- The **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.
  - Parametric test!
  - Null hypothesis: there is zero linear correlation between two variables.
  - Alternative hypothesis: there is [positive/negative/either] correlation between two variables.
- Spearman's Rank Test
  - Non-parametric test.
  - Behind the scenes - replaces actual data with their *rank*, computes the Pearson using ranks.
  - Same hypotheses.

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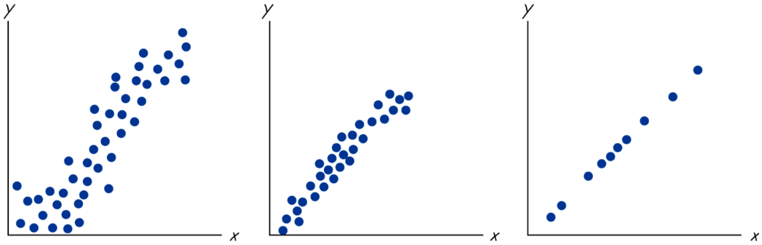
# Positive linear correlation






**(a)** Positive correlation between $x$ and $y$

**(b)** Strong positive correlation between $x$ and $y$

**(c)** Perfect positive correlation between $x$ and $y$

- Positive correlation: two variables move in the same direction.
- Stronger the correlation: closer the correlation coefficient is to 1.
- Perfect positive correlation: $\rho = 1$

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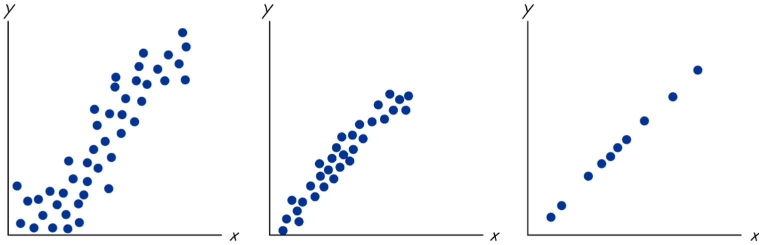


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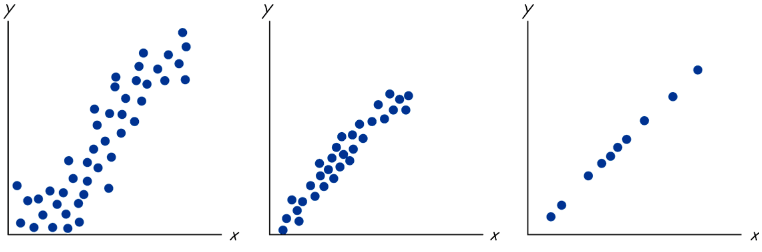


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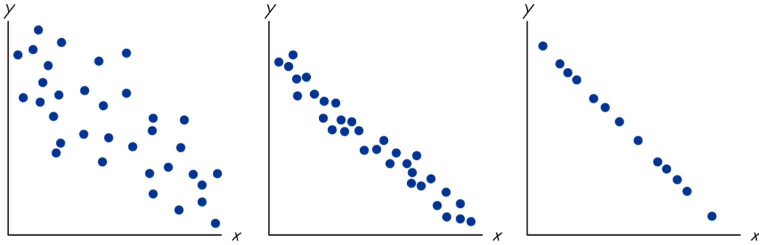


**(d)** Negative correlation between $x$ and $y$

**(e)** Strong negative correlation between $x$ and $y$

**(f)** Perfect negative correlation between $x$ and $y$

- Negative correlation: two variables move in opposite directions.
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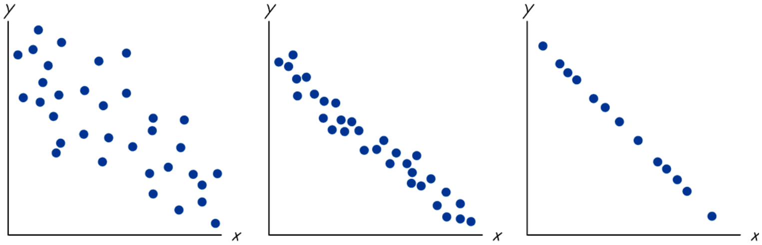


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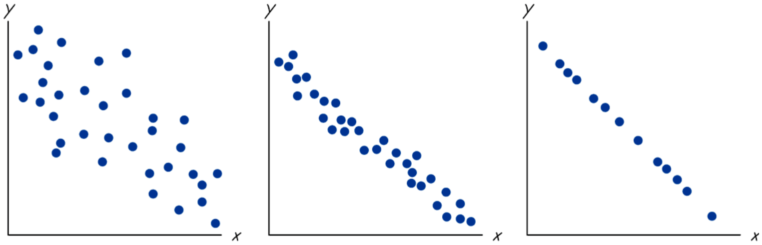


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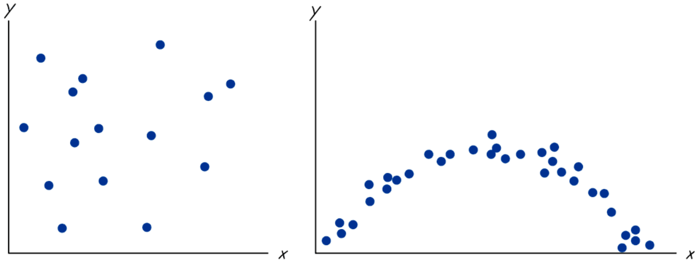


**(g)** No correlation between $x$ and $y$

**(h)** Nonlinear relationship between $x$ and $y$

- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
  - Cannot use regular correlation to detect this.

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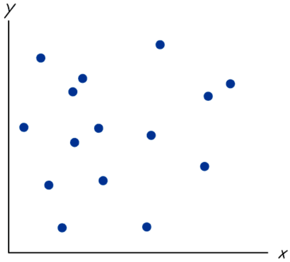


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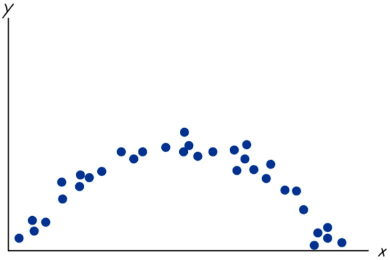


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# Example: Public Expenditure



- Data from 1960! about public expenditures per capita, and variables that may influence it:
  - Economic Ability Index
  - Percentage of people living in metropolitan areas.
  - Percentage growth rate of population from 1950-1960.
  - Percentage of population between the ages of 5-19.
  - Percentage of population over the age of 65.
  - Dummy variable: Western state (1) or not (0).
- Is there a statistically significant linear correlation between the percentage of the population who is young and the public expenditure per capita?
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# Regression



- Regression line: equation of the line that describes the linear relationship between variable $x$ and variable $y$.
- Need to assume that *independent variables* influence *dependent variables*.
  - $x$: *independent* or *explanatory* variable.
  - $y$: *dependent* variable.
  - Variable $x$ can influence the value for variable $y$, but not vice versa.
- Example: How does smoking affect lung capacity?
- Example: How does advertising affect sales?

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# Regression line



- Population regression line:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

- The actual coefficients $\beta_0$ and $\beta_1$ describing the relationship between $x$ and $y$ are unknown.
- Use sample data to come up with an estimate of the regression line:

$$y_i = b_0 + b_1 x_i + e_i$$

- Since $x$ and $y$ are not perfectly correlated, still need to have an error term.

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- Given a value for $x_i$, can come up with a **predicted value** for $y_i$, denoted $\hat{y}_i$.

$$\hat{y}_i = b_0 + b_1 x_i$$

- This is not likely be the actual value for $y_i$.
- **Residual** is the difference *in the sample* between the actual value of $y_i$ and the predicted value, $\hat{y}$.

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# Multiple Regression



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- Multiple regression line (sample):

$$y_i = b_0 + b_1 x_{1,i} + b_2 x_2 + ... + b_k x_k + e_i$$

  - $k$: number of parameters (coefficients) you are estimating.
  - $\epsilon_i$: error term, since linear relationship between the $x$ variables and $y$ are not perfect.
  - $e_i$: residual = the difference between the predicted value $\hat{y}$ and the actual value $y_i$.

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# Multiple Regression



- Multiple regression line (population):

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_2 + ... + \beta_{k-1} x_{k-1} + \epsilon_i$$

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## Least Squares Estimate



- How should we obtain the "best fitting line".
- Ordinary least squares (OLS) method.
- Choose sample estimates for the regression coefficients that minimizes:

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# Interpreting the slope



- Interpreting the slope, $\beta$: amount the $y$ is predicted to increase when increasing $x$ by one unit.
- When $\beta < 0$ there is a negative linear relationship.
- When $\beta > 0$ there is a positive linear relationship.
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- SPSS reports sample estimates for coefficients, along with...
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- Data from 1960 about public expenditures per capita, and variables that may influence it.
- In SPSS, choose `Analyze` menu and select `Regression` and `Linear`.
- Select `EX` (Expenditure per capita) as your dependent variable. This is the variable your are interested in explaining.
- Select your independent (aka explanatory) variables. These are the variables that you think can explain the dependent variable. I suggest you select these:
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# Sum of Squares Measures of Variation



- **Sum of Squares Regression (SSR)**: measure of the amount of variability in the dependent (Y) variable that is explained by the independent variables (X's).

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

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# Adjusted $R^2$



- $R^2$ will likely increase (slightly) even by adding nonsense variables.
- Adding such variables increases in-sample fit, but will likely hurt out-of-sample forecasting accuracy.
- The Adjusted $R^2$ penalizes $R^2$ for additional variables.

$$R^2_{\text{adj}} = 1 - \frac{n-1}{n-k-1}\left(1 - R^2\right)$$

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# F-test for Regression Fit



- F-test for Regression Fit: Tests if the regression line explains the data.
- Very, very, very similar to ANOVA F-test.
- $H_0 : \beta_1 = beta_2 = ... = \beta_k = 0$.
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# Assumptions from the CLT



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  - Useful for normality result from the Central Limit Theorem
  - Also necessary as you increase the number of explanatory variables.
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## Crucial Assumptions for Regression



- Linearity: a straight line reasonably describes the data.
  - Exceptions: experience on productivity, ordinal data like education level on income.
  - Consider transforming variables.
- Stationarity:
  - The central limit theorem: behavior of statistics as sample size approaches infinity!
  - The mean and variance must exist and be constant.
  - Big issue in economic and financial time series.
- Exogeneity of explanatory variables.
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## Multicollinearity



- **Multicollinearity:** when two or more of the explanatory variables are highly correlated.
- With multicollinearity, it is difficult to determine the effect coming from a specific individual variable.
- Correlated variables will have standard errors for coefficients will be large (coefficients will be statistically insignificant).
- Examples:
    - experience and age used to predict productivity
    - size of store (sq feet) and store sales used to predict demand for inventories.
    - parent's income and parent's education used to predict student performance.
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## Homoscedasticity



- **Homoscedasticity:** when the variance of the error term is constant (it does not depend on other variables).
- Counter examples (heteroscedasticity):
  - Impact of income on demand for houses.
  - Many economic and financial variables related to income suffer from this.
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