

Statistical Significance and Bivariate Tests

BUS 735: Business Decision Making and Research

1

1.1 Goals

Goals

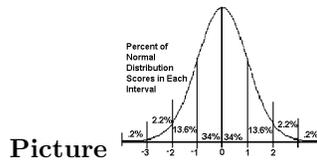
- Specific goals:
 - Re-familiarize ourselves with basic statistics ideas: sampling distributions, hypothesis tests, p-values.
 - Be able to distinguish different types of data and prescribe appropriate statistical methods.
 - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
 - LO1: Be able to construct and test hypotheses using a variety of bivariate statistical methods to compare characteristics between two populations.
 - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.

2 Statistical Significance

2.1 Sampling Distribution

Probability Distribution

- **Probability distribution:** summary of all possible values a variable can take along with the probabilities in which they occur.
- Usually displayed as:



Picture

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8129	0.8156	0.8183	0.8210	0.8236	0.8262
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749

Table

Formula

$$f(x|\mu, \sigma) =$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

- **Normal distribution:** often used “bell shaped curve”, reveals probabilities based on how many standard deviations away an event is from the mean.

Sampling distribution

- Imagine taking a sample of size 100 from a population and computing some kind of statistic.
- The statistic you compute can be anything, such as: mean, median, proportion, difference between two sample means, standard deviation, variance, or anything else you might imagine.
- Suppose you repeated this experiment over and over: take a sample of 100 and compute and record the statistic.
- A **sampling distribution** is the probability distribution *of the statistic*
- Is this the same thing as the probability distribution of the population?

Example

- Sampling Distribution Simulator:

http://www.stat.ucla.edu/dinov/courses_students.dir/Applets.dir/SamplingDistributionApplet.html

- In reality, you only do an experiment once, so the sampling distribution is a hypothetical distribution.
- Why are we interested in this?

Desirable qualities

What are some qualities you would like to see in a sampling distribution?

2.2 Central Limit Theorem

Central Limit Theorem

- Given:
 - Suppose a RV x has a distribution (it need not be normal) with mean μ and standard deviation σ .
 - Suppose a *sample mean* (\bar{x}) is computed from a sample of size n .
- Then, if n is sufficiently large, the sampling distribution of \bar{x} will have the following properties:
 - The sampling distribution of \bar{x} will be normal.
 - The mean of the sampling distribution will equal the mean of the population (consistent):

$$\mu_{\bar{x}} = \mu$$

- The standard deviation of the sampling distribution will decrease with larger sample sizes, and is given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem: Small samples

If n is small (rule of thumb for a single variable: $n < 30$)

- The sample mean is still consistent.
- Sampling distribution will be normal if the distribution of the population is normal.

Example 1

Suppose average birth weight is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.

What is the probability that a sample of size $n = 30$ will have a mean of $7.5lbs$ or greater?

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$
$$z = \frac{7.5 - 7}{1.5/\sqrt{30}} = 1.826$$

The probability the sample mean is greater than $7.5lbs$ is:

$$P(\bar{x} > 7.5) = P(z > 1.826) = 0.0339$$

Example 2

Suppose average birth weight is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.

What is the probability that a randomly selected baby will have a weight of 7.5lbs or more? What do you need to assume to answer this question? Must assume the population is normally distributed. Why?

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{7.5 - 7}{1.5} = 0.33$$

The probability that a baby is greater than 7.5lbs is:

$$P(x > 7.5) = P(z > 0.33) = 0.3707$$

Example 3

- Suppose average birth weight of all babies is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.
- Suppose you collect a sample of 30 newborn babies whose mothers used illegal drugs during pregnancy.
- Suppose you obtained a sample mean $\bar{x} = 6lbs$. If you assume the mean birth weight of babies whose mothers used illegal drugs has the same sampling distribution as the rest of the population, what is the probability of getting a sample mean this low or lower?

Example 3 continued

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{6 - 7}{1.5/\sqrt{30}} = -3.65$$

The probability the sample mean is less than or equal to 6lbs is:

$$P(\bar{x} < 6) = P(z < -3.65) = 0.000131$$

That is, if using drugs during pregnancy actually does still lead to an average birth weight of 7 pounds, there was only a 0.000131 (or 0.0131%) chance of getting a sample mean as low as six or lower. This is an extremely unlikely event if the assumption is true. Therefore it is likely the assumption is not true.

2.3 Hypotheses Tests

Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
 - Example: The population mean for systolic blood pressure is 120.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example about birth weights with mothers who use drugs.
 - Hypothesis: Using drugs during pregnancy leads to an average birth weight of 7 pounds (the same as with mothers who do not use drugs).

Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) *is equal to* some claimed value.
 - $H_0: \mu = 7$.
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter differs from the value given in the null hypothesis.
 - $H_a: \mu < 7$.
 - $H_a: \mu > 7$.
 - $H_a: \mu \neq 7$.
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an “innocent until proven guilty” policy.

Hypothesis tests

- (Many) hypothesis tests are all the same:

$$z \text{ or } t = \frac{\text{sample statistic} - \text{null hypothesis value}}{\text{standard deviation of the sampling distribution}}$$

- Example: hypothesis testing about μ :
 - Sample statistic = \bar{x} .
 - Standard deviation of the sampling distribution of \bar{x} :

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

P-values

- The P-value of the test statistic, is the area of the sampling distribution from the sample result in the direction of the alternative hypothesis.
- Interpretation: If the null hypothesis is correct, then the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence of the null hypothesis.
- The p-value is therefore a measure of *statistical significance*.
 - If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
 - If p-values are large, there is insignificant statistical evidence. When large, you fail to reject the null hypothesis.
- Best practice is writing research: report the p-value. Different readers may have different opinions about how small a p-value should be before saying your results are statistically significant.

3 Univariate Tests

3.1 Types of Data/Tests

Types of Data

- Nominal data: consists of categories that cannot be ordered in a meaningful way.
- Ordinal data: order is meaningful, but not the distances between data values.
 - Excellent, Very good, Good, Poor, Very poor.
- Interval data: order is meaningful, *and* distances are meaningful. However, there is *no natural zero*.
 - Examples: temperature, time.
- Ratio data: order, differences, and zero are all meaningful.
 - Examples: weight, prices, speed.

Types of Tests

- Different types of data require different statistical methods.
- Why? With interval data and below, operations like addition, subtraction, multiplication, and division are *meaningless!*

- Parametric statistics:
 - Typically take advantage of central limit theorem (imposes requirements on probability distributions)
 - Appropriate only for interval and ratio data.
 - More **powerful** than nonparametric methods.
- Nonparametric statistics:
 - Do not require assumptions concerning the probability distribution for the population.
 - There are many methods appropriate for ordinal data, some methods appropriate for nominal data.
 - Computations typically make use of data's *ranks* instead of actual data.

3.2 Hypothesis Testing about Mean

Single Mean T-Test

- Test whether the population mean is equal or different to some value.
- Uses the sample mean its statistic.
- Why T-test instead of Z-test?
- Parametric test that depends on results from Central Limit Theorem.
- Hypotheses
 - Null: The population mean is equal to some specified value.
 - Alternative: The population mean is [greater/less/different] than the value in the null.

Example: Public School Spending

- Dataset: average pay for public school teachers and average public school spending per pupil for each state and the District of Columbia in 1985.
- Download dataset `eduspending.xls`.
- Conduct the following exercises:
 - Show some descriptive statistics for teacher pay and expenditure per pupil.
 - Is there statistical evidence that teachers make less than \$25,000 per year?

- Is there statistical evidence that expenditure per pupil is more than \$3,500?

Using SPSS

Opening the data:

1. Save *eduspending.xls* somewhere.
2. Open SPSS.
3. Click radio button **Open an existing data source**.
4. Double-click **More files...**
5. Change **Files of type:** to **Excel**.
6. Go find and double click *eduspending.xls*.
7. Click **continue**.

Descriptive Statistics

1. Click **Analyze** menu, select **Descriptive Statistics**, then select **Descriptives**.
2. Click on **Pay** and click right arrow button.
3. Click on **Spending** and click right arrow button.
4. Click **Options**.
 - (a) Check any options you find interesting.
 - (b) Click **OK**
5. Click **OK**

Test Hypotheses

1. Click **Analyze** menu, select **Compare Means**, then select **One-Sample T test**.
2. Select **Pay**, then click right arrow.
3. Enter in **Test Value** text box 30000.
4. Output tables show descriptive statistics for pay, and hypothesis test results.

3.3 Hypothesis Testing about Proportion

Single Proportion T-Test

- **Proportion:** Percentage of times some characteristic occurs.
- Example: percentage of consumers of soda who prefer Pepsi over Coke.

$$\text{Sample proportion} = \frac{\text{Number of items that has characteristic}}{\text{sample size}}$$

Example: Economic Outlook

- Data from Montana residents in 1992 concerning their outlook for the economy.
- All data is ordinal or nominal:
 - AGE = 1 under 35, 2 35-54, 3 55 and over
 - SEX = 0 male, 1 female
 - INC = yearly income: 1 under \$20K, 2 20-35\$K, 3 over \$35K
 - POL = 1 Democrat, 2 Independent, 3 Republican
 - AREA = 1 Western, 2 Northeastern, 3 Southeastern Montana
 - FIN = Financial status 1 worse, 2 same, 3 better than a year ago
 - STAT = 0, State economic outlook better, 1 not better than a year ago
- Do the majority of Montana residents feel their financial status is the same or better than one year ago?
- Do the majority of Montana residents have a more positive economic outlook than one year ago?

Using SPSS

- Open the dataset *econoutlook.xls*.
- Parametric approach:
 1. If the variable is a zero or one, the *sample mean* is the same as the *proportion of the sample* that has a value equal to 1.
 2. Convert Financial Status (FIN) to a 0 if worse, and 1 if same or better:
 - (a) Click **Transform** menu, select **Recode into Different Variables**.
 - (b) Select FIN on left and click right arrow button.
 - (c) Click **Old and New Values** button.

- (d) First transform FIN=1 into value 0:
 - i. On the left under **Old Value**, click radio box for **Value**. In textbox enter 1.
 - ii. On the right under **New Value**, click radio box for **Value**. In textbox enter 0.
 - iii. Click **Add** button.
 - (e) Next transform FIN=2 or FIN=3 into value 1:
 - i. On the left under **Old Value**, click radio box for **Range**. In textboxes enter 2 and 3.
 - ii. On the right under **New Value**, click radio box for **Value**. In textbox enter 1.
 - iii. Click **Add** button.
 - (f) Click **Continue**
 - (g) In original Recode window, under **Output Variable**, type a name.
 - (h) Click **OK!**
3. Do a simple T-Test!
- Binomial approach:
 1. Independent observations that are a 0 or 1 have a binomial distribution (regardless of sample size).
 2. Click **Analyze** menu, select **Nonparametric Tests**, then select **Binomial**.
 3. On the left, select variable to test and click right arrow button.
 4. In Test Proportion, enter 0.5.
 5. Click **OK!**

3.4 Nonparametric Testing about Median

Single Median Nonparametric Test

- Why?
 - Ordinal data: cannot compute sample means (they are meaningless), only median is meaningful.
 - Small sample size and you are not sure the population is not normal.
- Sign test: can use tests for proportions for testing the median.
 - For a null hypothesized population median...
 - Count how many observations are above the median.
 - Test whether that proportion is greater, less than, or not equal to 0.5.
 - For small sample sizes, use binomial distribution instead of normal distribution.

Example

- Dataset: 438 students in grades 4 through 6 were sampled from three school districts in Michigan. Students ranked from 1 (most important) to 5 (least important) how important grades, sports, being good looking, and having lots of money were to each of them.
- Open dataset `gradschools.xls`. Choose second worksheet, titled `Data`.
- Answer some of these questions:
 - Is the median importance for grades is greater than 3?
 - Is the median importance for money less than 3?

Using SPSS to conduct nonparametric tests for medians

1. Click **Analyze** menu, select **Nonparametric Tests**, then select **Binomial...**
2. Click on **Grades** (or a different variable of interest), then click on right arrow.
3. Click radio button for **Cut point** and enter 3 into text box.
4. Do you want the exact (binomial distribution) p-value or asymptotic distribution (normal distribution)?
 - (a) Exact: click on **Exact...**
 - (b) Click **Exact** radio button.
 - (c) Click **Continue**.
5. Click **OK**
 - Output table shows exact p-value and normal distribution p-value for a two-tailed test.
 - What is your conclusion?

4 Bivariate Tests

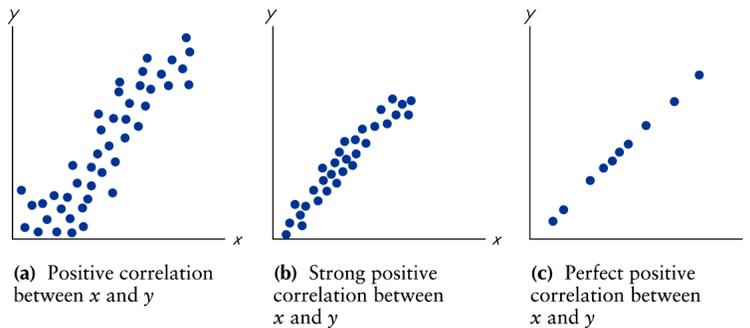
4.1 Correlation

Correlation

- A **correlation** exists between two variables when one of them is related to the other in some way.
- The **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.

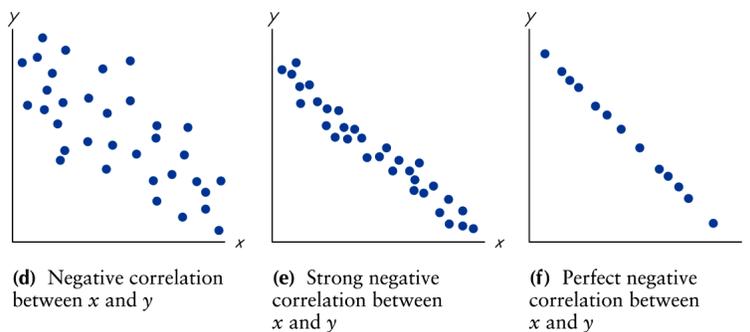
- Parametric test!
- Null hypothesis: there is zero linear correlation between two variables.
- Alternative hypothesis: there is [positive/negative/either] correlation between two variables.
- Spearman's Rank Test
 - Non-parametric test.
 - Behind the scenes - replaces actual data with their *rank*, computes the Pearson using ranks.
 - Same hypotheses.

Positive linear correlation



- Positive correlation: two variables move in the same direction.
- Stronger the correlation: closer the correlation coefficient is to 1.
- Perfect positive correlation: $\rho = 1$

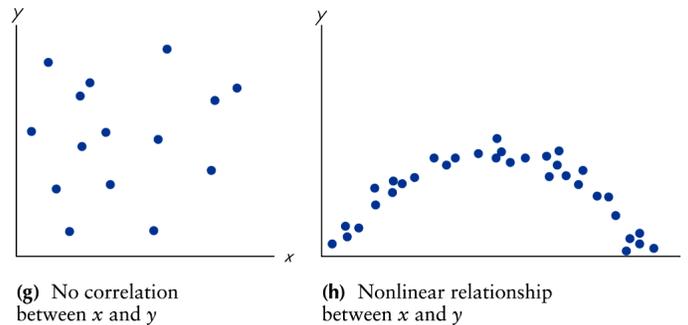
Negative linear correlation



- Negative correlation: two variables move in opposite directions.

- Stronger the correlation: closer the correlation coefficient is to -1.
- Perfect negative correlation: $\rho = -1$

No linear correlation



- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
 - Cannot use regular correlation to detect this.

Example: Public Expenditure

- Data from 1960! about public expenditures per capita, and variables that may influence it:
 - Economic Ability Index
 - Percentage of people living in metropolitan areas.
 - Percentage growth rate of population from 1950-1960.
 - Percentage of population between the ages of 5-19.
 - Percentage of population over the age of 65.
 - Dummy variable: Western state (1) or not (0).
- Is there a statistically significant linear correlation between the percentage of the population who is young and the public expenditure per capita?
- Is there a statistically significant linear correlation between the public expenditure per capita and whether or not the state is a western state?

Using SPSS

1. Open the dataset *publicexp.xls* in SPSS.
2. For a parametric test (Pearson correlation):
3. Select **Analyze** menu, select **Correlate**, then select **Bivariate**.

4. Select at least two variables (it will do all pairwise comparisons) on the left and click right arrow button.
5. Select check-box for **Pearson** and/or **Spearman**.
6. Click OK!

4.2 Difference in Populations (Independent Samples)

Difference in Means (Independent Samples)

- Suppose you want to know whether the mean from one population is larger than the mean for another.
- Examples:
 - Compare sales volume for stores that advertise versus those that do not.
 - Compare production volume for employees that have completed some type of training versus those who have not.
- Statistic: Difference in the sample means ($\bar{x}_1 - \bar{x}_2$).

Independent Samples T-Test

- Hypotheses:
 - Null hypothesis: the difference between the two means is zero.
 - Alternative hypothesis: the difference is [above/below/not equal] to zero.
- Different ways to compute the test depending on whether...
 - the variance in the two populations is the same (more powerful test), or...
 - the variance of the two populations is different.
 - To guide you, SPSS also reports Levene's test for equality of variance (Null - variances are the same).

Example

- Dataset: average pay for public school teachers and average public school spending per pupil for each state and the District of Columbia in 1985.
- Test the following hypotheses:
 - Does spending per pupil differ in the North (region 1) and the South (region 2)?

- Does teacher salary differ in the North and the West (region 3)?
- Do you see any weaknesses in our statistical analysis?

Using SPSS to test differences in means

1. Open *eduspending.xls* in SPSS.
2. Click **Analyze** menu, select **Compare Means**, then select **Independent-Samples T test**.
3. Select **Pay** or **Spending**, depending on which you are currently interested in.
4. Click the right arrow that is just to the left of **Test Variables**.
5. Select **Area** and click on right arrow to the left of **Grouping Variable**.
6. You need to tell SPSS what your grouping variable means and what groups you are interested in:
 - (a) Click on **Define Groups**
 - (b) Click radio button **Use specified values**.
 - (c) Enter in the appropriate numbers for Group 1 and Group 2 (i.e. if you want the North to be group 1, type a 1 in Group 1 text box, and if you want the West to be group 2, type a 3 in the Group 2 text box.
 - (d) Click **Continue**.
7. Click **OK!**
 - The first output table shows some descriptive statistics for each group.
 - The next output table shows:
 - Statistical evidence about whether the variances are different.
 - Statistical evidence about whether the means are different.
 - Descriptive statistics about the difference in the means.
 - Confidence intervals for the difference in the means.

Nonparametric Tests for Differences in Medians

- Mann-Whitney U test: nonparametric test to determine difference in *medians*.
- Assumptions:
 - Samples are independent of one another.

- The underlying distributions have the same shape (i.e. only the location of the distribution is different).
- It has been argued that violating the second assumption does not severely change the sampling distribution of the Mann-Whitney U test.
- Null hypothesis: medians for the two populations are the same.
- Alternative hypotheses: medians for the two populations are different.

Using SPSS to conduct nonparametric tests for medians

1. Click **Analyze** menu, select **Nonparametric Tests**, then select **2 Independent Samples...**
 2. Move **Pay** (or whatever you are interested in) into **Test Variable List**
 3. Move **Area** into **Grouping Variable**
 4. Define groups as before.
 5. You can get exact p-values if absolutely necessary (takes more time).
 6. Click **OK**.
- The value of the Mann-Whitney test statistic can be huge and this does not have much interpretation (it's equal the smaller of the two sums of ranks).
 - The Significance is the p-value.
 - What is your conclusion?

4.3 Paired Samples

Dependent Samples - Paired Samples

- Use a **paired sampled test** if instead the two samples have the same individuals before and after some treatment.
- Really simple: for each individual subtract the before treatment measure from the after treatment measure (or vice-versa).
- Treat your new series as a single series.
- Conduct one-sample tests.
- In SPSS, you need to have separate columns for each of these variables.
- There are methods in SPSS specifically for Dependent Samples tests - but the paired sampled approaches are equivalent to one-sample tests.

5

Conclusions

- Ideas to keep in mind:
 - What is a sampling distribution? What does it imply about p-values and statistical significance?
 - When it is appropriate to use parametric versus non-parametric methods.
 - Most univariate and bivariate questions have a parametric and non-parametric approach.
- Homework:
 - Read all of chapter 1.
 - Problem handed out in class.
 - Chapter 1 end of chapter exercises (these may include hypothesis tests we did not cover!), Part I and II.
 - Start chapter 3.
- Next: Regression Analysis - looking at more complex relationships between more than 2 variables.