Linear Programming

BUS 735: Business Decision Making and Research

Tuesdsay, November 27, 2012

1 Goals and Agenda

Learning Objective	**Active Learning Activity**
Learn how to formulate optimization problems with constraints (linear programming problems).	Lecture / Discussion
Learn how to formulate and show a graphical solution linear programming problems.	Worksheet problem.
Learn how to solve linear programming problems using Excel	Re-solve same worksheet problem with Excel.
Practice makes perfect!	Another worksheet problem.
Access what we have learned.	Quiz on linear programming.
Learn the value of easing constraints.	Lecture/Discussion and worksheet problem.
Practice makes perfect!	Another worksheet problem.

1.1 Goals

Goals of this class meeting

- Specific Goals:
  - Learn how to set up problems with constraints.
  - Learn how to maximize or minimize objectives subject to constraints.
  - Learn how to solve linear programming problems in Excel.
- Learning Objectives:
  - LO4: Be able to construct and solve linear programming models to answer business optimization problems.
  - LO7: Have a sound familiarity of various statistical and quantitative methods in order to be able to approach a business decision problem and be able to select appropriate methods to answer the question.

2 Linear Programming

2.1 Objectives and Constraints

Linear Programming

- Linear programming: problems that involve *linear* objective functions and *linear* constraints.
- Possible objectives:
  - maximize profits, minimize costs, maximize revenues, minimize time to complete projects, minimize the total distances needed to transport goods from warehouses to stores, minimize total energy consumption.
- Objective function is subject to constraints such as:
  - Limited resources, restrictive guidelines, demand for goods at stores, supply of goods at warehouses, energy requirements for machines or buildings, time requirements for completing intermediate steps of a project.

2.2 Assembly Lines

Assembly Lines

Suppose a company produces two products that are processed on two assembly lines. Assembly line 1 has 100 available hours, and assembly line 2 has 42 available hours. Each product requires 10 hours on assembly line 1, while Product A needs 7 hours on assembly line 2, and Product B needs 3 hours on assembly line 2. The company earn \$6 profit for every item of Product A produced and \$4 profit for every item of Product B produced.

Worksheet questions

1. What is the company's objective?

2. What are the company's choice variables? Denote these x_1, x_2, etc.

3. What is the company's objective function?

4. What are the company's constraints? What's stopping them from producing infinite amounts of of each product? Write down the functional relationships for the constraints. Did you use equal signs or inequalities?

5. Graph the objective function if profit is equal to 30 (this could be called an *iso-profit function*).

6. On the same graph, plot the iso-profit function for profit equal to 40.

7. On the same graph, plot the iso-profit function for profit equal to 50.

8. What do you notice about the graphs?

9. On a new graph, plot the constraints and shade the **feasibility region**. The feasibility region is the set of all decisions that satisfy all constraints.

10. Understanding the behavior of iso-profit functions, show on your graph the possible candidates for the decision variables that maximize the company's objective?

11. Compute exactly the quantities for the decision variables.

12. What decision maximizes the company's objective?

13. Use Excel to compute this answer.
    (a) Enter and label the decision variables.
    (b) Compute profit from these decision variables.
    (c) Enter and label parameters for constraints.
    (d) Compute usage for each constraint.
    (e) Enter and label the maximum values for each constraint.
    (f) Use the Solver tool - this comes with Excel, no special software is necessary.

2.3 Copperfield Mining Company

Copperfield Mining Company

Copperfield Mining Company owns two mines, each of which produces three grades of ore - high, medium, and low. The company has a contract to supply a smelting company with at least 12 tons of high-grade ore, 8 tons of medium-grade ore, and 24 tons of low-grade ore. Mine 1 produces 6 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. Mine 2 produces 2 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. The cost of operating mines is \$200 for mine 1 and \$160 for mine 2.

Worksheet questions

1. What is the company's objective?

2. What are the company's choice variables? Denote these x_1, x_2, etc.

3. What is the company's objective function?

4. What are the company's constraints? What's stopping them from producing infinite amounts of of each product? Write down the functional relationships for the constraints.

5. Plot the constraints and shade the **feasibility region**. The feasibility region is the set of all decisions that satisfy all constraints.

6. Use Excel to compute this answer.

Quiz!

A California grower has a 50 acre farm on which to plant strawberries and tomatoes. The grower has available 300 hours of labor per week and 800 tons of fertilizer. Each acre of strawberries requires 10 hours of labor and 8 tons of fertilizer, whereas each acre of tomatoes requires 3 hours of labor and 20 tons of fertilizer. The farmer earns $400 profit for each acre of strawberries and $300 profit for each acre of tomatoes. The farmer wants to maximize his profits.

1. (20% weight) What is the farmer's objective function?

2. (30% weight) What are the farmer's constraints?

3. (30% weight) Plot the farmer's constrains and shade the feasibility region. Show the points on the graph that are candidates for the optimal solution.

4. (20% weight) Use Excel to compute the farmers optimal solution. How much of each crop does the farmer produce? What are her total profits at this point?

3 Shadow Prices

3.1 Shadow Price Examples

Shadow Prices

- **Shadow Prices**: aka "dual prices", aka "marginal values", aka "Lagrange multipliers", is the price the decision maker would be willing to pay for a marginal ease in one of the constraints.
- Example from Assembly lines: what is the maximum price the assembly line manager be willing to pay for one extra allowable hour for Assembly line A?
- Example from Mining Company: how much would the mining company save if it could produce one less ton of low-grade iron ore?
- Example from Farmers on quiz: how much would the farmer be willing to pay for one additional hour of labor?
- Be careful with shadow prices: they assume one and only one change is happening. They are not valid with more than one change.

Worksheet questions

3.2 Example: Farmer from the Quiz

1. Compute the solution for the Farmer's profit maximization problem on the quiz. Select `Answers Report` and `Sensitivity Report`.
2. On the sensitivity report, what is shadow price of fertilizer?

3. Copy and *Paste As Value* the maximum profit earned, label this "Original profit".
4. Below this compute the difference between profit and original profit, right now this should equal zero.
5. Change the constraint on the fertilizer to 801 tons and recompute the solution.

6. How much did profit increase by?

7. Make an educated guess on how much profit would increase if the constraint on fertilizer was 900 tons.

8. Change the constraint on the fertilizer to 900 tons and recompute the solution. How much larger is profit compared to the original solution?

9. Make an educated guess on how much profit would increase if the constraint on fertilizer was 2800 tons.

10. Change the constraint on the fertilizer to 2800 tons and recompute the solution.

11. How much larger is profit compared to the original solution?

12. Why was your guess incorrect?

3.3 Example: Bluegrass Distillery

Bluegrass Distillery

Bluegrass Distillery produces a custom-blend whiskey make from rye and bourbon. The company has received an order for a minimum of 400 gallons. Manufacturing constraints limit the distillery from making more than 500 gallons. The customer has specified that the bourbon should contain at least 40% rye and not more than 250 gallons of bourbon. The blend is sold for $5 per gallon, and the company's costs are $2 per gallon of rye and $1 per gallon of bourbon. The company wants to determine a mix than will maximize its profits.

Worksheet questions

1. What is the company's objective? What are its decision variables.

2. What is the company's objective function?

3. What are its constraints?

4. Use Excel to compute the optimal solution. Select the `Answer` and `Sensitivity` reports.

5. Look at the answer report. Are all of the constraints binding? If not, which constraints are not binding. What are those constraints and what is the company's usage?

6. Without calculating or looking at anything, what should the shadow prices be for the non-binding constraints?

7. What are the shadow prices for the binding constraints? What is the interpretation? That is, if you were advising someone at the distillery who knew nothing about linear programming, how would you explain the shadow prices?