Simulation and Queing Theory

BUS 735: Business Decision Making and Research

Goals and Agenda

Learning Objective	**Active Learning Activity**
Learn how to simulate probability distribution	Example problem in Excel
Learn how to simulate inventory systems.	Example problem in Excel
Learn how to simulate queuing systems.	Two example problems in Excel
Practice makes perfect	Exercises in Excel
More practice	Read Chapter 12, pp 488-508

Simulating Probability Distributions

- **Simulation:** drawing random numbers from a probability distribution.
- **Monte Carlo Simulation:** Use simulated data to simply compute means, standard deviations, etc.
- More complicated computations can be made based on the simulated data.
  - Create linear combinations of variables.
  - Take ratios!

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Example

- Suppose the MacGuys sell somewhere between 0 and 4 computers each week from their store, according to the probability distribution to the right.
- Simulate computer demand
- Computers sell for $4,300 each.
- Compute the mean and standard deviation for weekly demand for computers.
- Compute the mean and standard deviation for weekly revenue.

Probability Distribution:

Demand	Prob.
0	0.2
1	0.4
2	0.2
3	0.1
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Something More Complicated

- Suppose there is an inventory cost of $50 per computer.
- If the company falls short, the company not only fails to make a sale, but is estimated to loose $500 in future revenue per computer, due to making a customer unhappy.
- Suppose the company orders 1 computer per week.
- Simulate demand for two years (104 weeks), simulate inventory, shortage, and surplus, and revenue

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Inventory System of Equations

Simulate inventory, sales, surplus, and shortage:

$$\text{Inventory}_t = \text{Inventory}_{t-1} - \text{Sales}_{t-1} + 1$$

$$\text{Sales}_t = \min(\text{Inventory}_t, \text{Demand}_t)$$
$$\text{Surplus}_t = \text{Inventory}_t - \text{Sales}_t$$
$$\text{Shortage}_t = \text{Demand}_t - \text{Sales}_t$$

Simulate revenue with \$50 inventory cost, \$500 shortage cost:

$$\text{Revenue}_t = (\$4,300)\ \text{Sales}_t - (\$50)\ \text{Inventory}_t - (\$500)\text{Shortage}_t$$

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Queuing System Example: Denim Factory

- A denim manufacturing facility receives yarn at varying time intervals (according to the probability distribution in the following slide).
- Then it dyes the yarn, which takes varying amounts of time according to the second probability distribution (according to the second probability distribution on the following slide).
- If a batch of yarn arrives at the facility, it is possible it must wait for the previous batch to complete.
- It is possible that facility sits not utilized while it waits for another batch of yarn to arrive.
- Calculate the mean and std dev for the total time in the facility (waiting time + dying time).
- Calculate the mean and std dev for the waiting time.
- Calculate the average number of days per month the facility is idle.

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Queuing System Probability Distributions

Distribution of Arrival Intervals:

Arrival Interval	Probability
1 day	0.2
2 days	0.4
3 days	0.3
4 days	0.1

Distribution of Dying Times:

Dying Time	Probability
0.5 days	0.2
1 day	0.5
2 days	0.3

Queuing System Equations

Compute the following:

1. Simulate Interval_i.
2. $\text{Arrival}_i = \text{Arrival}_{i-1} + \text{Interval}_i$
3. $\text{Waiting}_i = max(\text{Finish}_{i-1} - \text{Arrival}_i, 0)$
4. $\text{Idle}_i = max(\text{Arrival}_i - \text{Finish}_{i-1}, 0)$
5. Simulate Dying_i.
6. $\text{TimeSystem}_i = \text{Waiting}_i + \text{Dying}_i$
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Queuing System Example: Bank Teller

A bank is trying to determine whether it should install one or two drive-through teller windows.

Dist Arrival Intervals:

Arrival Interval	Probability
1 min	0.2
2 min	0.6
3 min	0.1
4 min	0.1

Dist Service Times:

Service Time	Probability
2 min	0.1
3 min	0.4
4 min	0.2
5 min	0.2
5 min	0.1

- Assume customers enter shorter line, in case of a tie randomly pick a line with equal probability
- Simulate a one-teller system
- Simulate a two-teller system
- Compute average queue length, waiting time, utilization

Queuing System Equations

1. Simulate Interval_i
2. $\text{Arrival}_i = \text{Arrival}_{i-1} + \text{Interval}_i$
3. $\text{Wait1}_i = \max\left[\max(\text{TimeOut1}_0 : \text{TimeOut1}_{i-1}) - \text{Arrival}_{i-1}, 0\right]$
4. $\text{Wait2}_i = \max\left[\max(\text{TimeOut2}_0 : \text{TimeOut2}_{i-1}) - \text{Arrival}_{i-1}, 0\right]$
5. $\text{Idle1}_i = \max\left[\text{Arrival}_{i-1} - \max(\text{TimeOut1}_0 : \text{TimeOut1}_{i-1}), 0\right]$
6. $\text{Idle2}_i = \max\left[\text{Arrival}_{i-1} - \max(\text{TimeOut2}_0 : \text{TimeOut2}_{i-1}), 0\right]$
7. $\text{Length1}_i = \text{COUNTIF}(\text{TimeOut1}_0 : \text{TimeOut1}_{i-1}, '' >'' \&\text{Arrival}_i)$
8. $\text{Length2}_i = \text{COUNTIF}(\text{TimeOut2}_0 : \text{TimeOut2}_{i-1}, '' >'' \&\text{Arrival}_i)$
9. LineChoice_i = nested IF() to compare Length1_i and Length2_i
10. $\text{ActualWait}_i = \text{IF}(\text{LineChoice}_i = 1, \text{Wait1}_i, \text{Wait2}_i)$
11. Simulate ServiceTime_i
12. $\text{TotalTime}_i = \text{ActualWait}_i + \text{ServiceTime}_i$
13. $\text{TimeOut1}_i = \text{IF}(\text{LineChoice}_i = 1, \text{Arrival}_i + \text{TotalTime}_i, 0)$
14. $\text{TimeOut2}_i = \text{IF}(\text{LineChoice}_i = 2, \text{Arrival}_i + \text{TotalTime}_i, 0)$

Exercise: Milk Demand

- A local organic grocery store orders 16 cases of milk from a dairy on a weekly basis.
- Store pays $10 per case of milk
- Store sells milk for $16 per case
- Inventory cost is $0.50 per case per week
- Shortage cost is $1 per case per week

Probability Distribution:

Demand	Prob.
15	0.20
16	0.25
17	0.40
18	0.15

Simulate inventory, sales, shortages, surpluses, revenue, costs, and profits for two years, and report averages.

Exercise: Oil Tanker Queue

Oil tankers arrive at a single loading dock at random intervals and the time is takes to fill a tanker with oil and prepare it for sea is randomly determined.

Dist Arrival Intervals:

Arrival Interval	Probability
1 day	0.05
2 days	0.10
3 days	0.20
4 days	0.3
5 days	0.2
6 days	0.1
7 days	0.05

Service Times:

Service Time	Probability
3 days	0.1
4 days	0.2
5 days	0.4
6 days	0.3

Simulate movement of 400 tankers to and from the loading dock and compute the average wait time and queue length.