Measuring Interest Rates

Economics 301: Money and Banking
Goals:

- Learn to compute present values, rates of return, rates of return.

Learning Outcomes:

- LO3: Predict changes in interest rates using fundamental economic theories including present value calculations, behavior towards risk, and supply and demand models of money and bond markets.
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Read Hubbard and O’Brien, Chapter 3.
Cash flows:

Cash flows: size and timing of payments made for various debt instruments.

Present value:

Present value: aka present discounted value, discounts payments made in the future to a current date equivalent.

Present value depends on assumption for interest rate.

- Higher interest rates - higher degree of discount.
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Simple Loan Example

- Simple loan: lender provides funds to borrower, borrower pays back principal and interest at maturity date.
- Suppose interest rate is 5% (denote with \( i \)), simple loan of $100 (denote with \( P \)).
- Balance (denote with \( A \)) with a one year maturity:
  - \( A_1 = P(1 + i) = 100(1 + 0.05) = 105 \).
- Let it ride for another year...
  - \( A_2 = A_1(1 + i) = 105(1 + 0.05) = 110.25 \).
  - \( A_2 = P(1 + i)(1 + i) = P(1 + i)^2 = 100(1 + 0.05)^2 = 110.25 \).
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Present Value

- Present value: indifferent between $100 today, $105 next year, or $110.25 in two years.
- Given future cash flow of $105 or $110.25, respectively, the present value is,

\[ PV = 100 = \frac{105}{(1 + 0.05)} \]

\[ PV = 100 = \frac{110.25}{(1 + 0.05)^2} \]

- General formula,

\[ PV = \frac{CF_n}{(1 + i)^n} \]

- Example: what is the present value of $100,000 to be paid in 30 years if the interest rate is 4%?
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Types of Credit Market Instruments

- Simple loan.
- Fixed-payment loan: borrower makes a fixed payment (that includes interest and principal) each period until maturity date.
- Coupon bond: borrower pays fixed interest payments (coupon payments) until maturity date, pays face value at maturity.
  - Coupon rate: dollar amount of coupon payments as a percentage of face value. Related to, but not exactly an interest rate.
- Discount bond: bought at a price below its face value, makes no payments until maturity date, at which time pays face value.
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Compounded interest: when interest payments are made multiple times in a given period.

- Compounded annually: full interest payment paid out once per year.
- Compounded quarterly: payment for 1/4 of interest rate made 4 times per year.
- Compounded monthly: payment for 1/12 of interest rate made 12 times per year.
- Compounded daily: payment for 1/365 of interest rate made 365 times per year.
- Compounded continuously: interest payments constantly made. Occurs in nature.
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Present Value Computations

- Present value of a stream of cash flows \((CF_t)\) from time \(t = 0\) (today) to \(t = T\):

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PV = \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t} = CF_0 + \frac{CF_1}{1+i} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_T}{(1+i)^T}
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- Suppose you have an auto loan,
  - Annual interest rate is 6% interest.
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  - Five year loan.
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The geometric series is a useful mathematical tool in PV computations: If $\beta \in (0, 1)$, then,

$$\frac{1}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \beta^4 + \ldots$$

- Used in present values: $\beta = \frac{1}{1+i}$, which is between 0 and 1 for positive interest rates.
- Used for cash flows that occur every period forever. Eg: Perpetuity, stock dividends?
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Multiply present value $1/(1 - \beta)$ (previous slide) by $\beta^T$:

$$\frac{\beta^T}{1 - \beta} = \beta^T + \beta^{T+1} + \beta^{T+2} + \beta^{T+3} + \ldots$$

Subtract this equation from $1/(1 - \beta)$ (previous slide):

$$\frac{1 - \beta^T}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \ldots + \beta^{T-1}$$

Used for cash flows that begin in current period (0) through period T-1.

For cash flows beginning in period $s$ and lasting through period $T$:

$$\frac{\beta^s - \beta^{T+1}}{1 - \beta} = \beta^s + \beta^{s+1} + \beta^{s+2} + \ldots + \beta^T$$
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Compute the present value of coupon bond with
- Face value $3000.
- 10 year maturity.
- Coupon rate 6%.
- Annual payment beginning in one year.
- Prevailing interest rate 5%.

Compute the present value of a discount bond with,
- Face value $5000.
- 5 year maturity.
- Prevailing interest rate 8%.
More Computations

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Yield to Maturity

- **Yield to maturity**: the annual interest rate that equates the present value of cash flow of payments received from a debt instrument with its current day value.

- Example: yield to maturity for a simple loan.
  - $PV = \text{Cash borrowed} = \$200$.
  - $CF = \text{Cash flow} = \text{payment received after } n = 5 \text{ years} = \$280.51$.

\[
PV = \frac{CF}{(1 + i)^n} \rightarrow 200 = \frac{280.51}{(1 + i)^5}
\]

\[
(1 + i)^5 = \frac{280.51}{200} \rightarrow 1 + i = \left(\frac{280.51}{200}\right)^{\frac{1}{5}}
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\[
1 + i = 1.07 \rightarrow i = 7\%
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(1 + i)^5 = \frac{280.51}{200} \quad \rightarrow \quad 1 + i = \left( \frac{280.51}{200} \right)^{\frac{1}{5}}
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- Present value of a coupon bond for,
  - Coupon payment = \( CF \).
  - Face value = \( F \).
  - Years to maturity = \( T \).

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Rate of Return

- **Rate of return**: the total benefits received from holding a security, expressed as a percentage of purchase price.
- Rate of return includes interest payments *plus capital gains*.
- Rate of return for holding a bond from time $t$ to $t + 1$ is,
  \[ R = \frac{CF + P_{t+1} - P_t}{P_t} \]
  - $R$: rate of return.
  - $P_t$: price of bond at time $t$.
- Can also express rate of return as the sum, $R = i + g$, where,
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- purchased for $1,500,
- makes a single interest payment of $100,
- sold for $1,600.

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Long-term debt instruments have a high degree of interest rate risk.

**interest rate risk**: changes in interest rates over the life of the debt instrument influence the secondary market price of the bond, influencing capital gains and therefore rate of return.

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Analyzing behavior of interest rates and asset markets using supply and demand model.

Reading: Chapter 4.