

# Interest Rates, Cash Flows, and Rates of Return

ECO 301: Money and Banking

# Goals and Learning Outcomes

- Goals:
  - Learn to compute present values, rates of return, rates of return.
- Learning Outcomes:
  - LO3: Predict changes in interest rates using fundamental economic theories including present value calculations, behavior towards risk, and supply and demand models of money and bond markets.

# Reading and Exercises

- Present values and future values: Chapter 3, pp. 55-63
- Debt instruments: Chapter 3, pp. 63-67
- Yield to maturity: Chapter 3, pp. 68-78
- Rates of return: Chapter 3, pp. 79-83
- **Canvas quiz due Wed 11:59 PM.**
- **Homework/Exercise due Fri 11:59 PM.** We will work together in class on Thursday

# Cash Flows

- **Cash flows:** size and timing of payments made for various debt instruments.
- **Present value:** aka **present discounted value**, discounts payments made in the future to a current date equivalent.
- Present value depends on assumption for interest rate.
  - Higher interest rates - higher degree of discount.

# Simple Loan Example

- Simple loan: lender provides funds to borrower, borrower pays back principal and interest at maturity date.
- Suppose interest rate is 5% (denote with  $i$ ), simple loan of \$100 (denote with  $P$ ).
- Balance (denote with  $A$ ) with a one year maturity:
  - $A_1 = P(1 + i) = \$100(1 + 0.05) = \$105$ .
- Let it ride for another year...
  - $A_2 = A_1(1 + i) = \$105(1 + 0.05) = \$110.25$
  - $A_2 = P(1 + i)(1 + i) = P(1 + i)^2 = \$100(1 + 0.05)^2 = \$110.25$
- At the end of  $n$  years, we have
  - $A_n = P(1 + i)^n$ .

# Present Value

- Present value: indifferent between \$100 today, \$105 next year, or \$110.25 in two years.
- Given future cash flow of \$105 or \$110.25, respectively, the present value is,

$$PV = 100 = \frac{105}{(1 + 0.05)}$$

$$PV = 100 = \frac{110.25}{(1 + 0.05)^2}$$

- General formula,

$$PV = \frac{CF_n}{(1 + i)^n}$$

- Example: what is the present value of \$100,000 to be paid in 30 years if the interest rate is 4%?

# Types of Credit Market Instruments

- Simple loan.
- Fixed-payment loan: borrower makes a fixed payment (that includes interest and principal) each period until maturity date.
- Coupon bond: borrower pays fixed interest payments (coupon payments) until maturity date, pays face value at maturity.
  - **Coupon rate:** dollar amount of coupon payments as a percentage of face value. Related to, but not exactly an interest rate.
- Discount bond (aka Zero-Coupon Bond): bought at a price below its face value, makes no payments until maturity date, at which time pays face value.
- Often in finance practice and disciplines, *discount bond* refers to any bond that is purchased below its face value.

# Compounded Interest

- **Compounded interest:** when interest payments are made multiple times in a given period.
- Compounded annually: full interest payment paid out once per year.
- Compounded quarterly: payment for  $1/4$  of interest rate made 4 times per year.
- Compounded monthly: payment for  $1/12$  of interest rate made 12 times per year.
- Compounded daily: payment for  $1/365$  of interest rate made 365 times per year.
- Compounded continuously: interest payments constantly made. Occurs in nature.

# Present Value Computations

- Present value of a stream of cash flows ( $CF_t$ ) from time  $t = 0$  (today) to  $t = T$ :

$$PV = \sum_{t=0}^T \frac{CF_t}{(1+i)^t} = \frac{CF_1}{1+i} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_T}{(1+i)^T}$$

- Example: Auto Loan
  - Annual interest rate is 6% interest.
  - Compounded monthly, so  $i = 0.06/12 = 0.005$ .
  - Five-year loan, so  $T = 5 * 12 = 60$ .
  - Monthly payment, so  $CF = \$200$ .
  - Present value is amount you borrow today.

# Present Value Computations

- The geometric series is a useful mathematical tool in PV computations: If  $\beta \in (0, 1)$ , then,

$$\frac{1}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \beta^4 + \dots$$

- Used in present values:  $\beta = \frac{1}{1+i}$  which is between 0 and 1 for positive interest rates.
- Used for cash flows that occur every period forever.  
Eg: Perpetuity, stock dividends?

# Present Value Calculations

- Multiply present value  $1/(1 - \beta)$  (previous slide) by  $\beta^T$ :

$$\frac{\beta^T}{1 - \beta} = \beta^T + \beta^{(T+1)} + \beta^{(T+2)} + \beta^{(T+3)} + \dots$$

Subtract the this equation from  $1/(1 - \beta)$  (previous slide):

$$\frac{1 - \beta^T}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \dots + \beta^{T-1}$$

Used for cash flows that begin in current period (0) through period T-1

- For cash flows beginning in period  $s$ , with  $T$  number of periods:

$$\frac{\beta^s - \beta^{T+s}}{1 - \beta} = \beta^s + \beta^{s+1} + \beta^{s+2} + \dots + \beta^{s+T-1}$$

# More Computations

Compute the present value of coupon bond with

- Face value \$3000
- 10 year maturity
- Semi-annual payments
- Coupon rate 6%
- Annual payment beginning in one year
- Prevailing annual interest rate 5%

Compute the present value of a zero-coupon bond with,

- Face value \$5000
- 5 year maturity
- Prevailing interest rate 8%

# Yield to Maturity

- **Yield to maturity:** the annual interest rate that equates the present value of cash flow of payments received from a debt instrument with its current day value.
- Example: yield to maturity for a simple loan.
  - PV = Cash borrowed = \$200.
  - CF = Cash flow = payment received after  $n = 5$  years \$280.51.

$$PV = \frac{CF}{(1+i)^n} \quad \rightarrow \quad 200 = \frac{280.51}{(1+i)^5}$$

$$(1+i)^5 = \frac{280.51}{200} \quad \rightarrow \quad 1+i = \left(\frac{280.51}{200}\right)^{\frac{1}{5}}$$

$$1+i = 1.07 \quad \rightarrow \quad i = 7\%$$

# Yield to Maturity: Coupon bond

- Present value of a coupon bond for,
  - Coupon payment =  $CF$ .
  - Face value =  $F$ .
  - Years to maturity =  $T$ .

$$PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \dots + \frac{CF}{(1+i)^T} + \frac{F}{(1+i)^T}$$

$$PV = \sum_{t=1}^T \frac{CF}{(1+i)^t} + \frac{F}{(1+i)^T}$$

- To find yield to maturity, solve for  $i$ . Impossible to do algebraically → use financial calculator.

# Rate of Return

- **Rate of return:** the total benefits received from holding a security, expressed as a percentage of purchase price.
- Rate of return includes interest payments *plus capital gains*.
- Rate of return for holding a bond from time  $t$  to  $t + 1$  is,

$$R = \frac{CF + P_{t+1} - P_t}{P_t}$$

–  $R$ : rate of return,  $P_t$ : price of bond at time  $t$

- Can also express rate of return as the sum,  $R = i + g$ , where,

$$\text{rate of capital gain} = g = \frac{P_{t+1} - P_t}{P_t},$$

$$\text{interest rate} = i = \frac{CF}{P_t}$$

# Rate of Return

- Suppose a debt instrument is held for one year that is,
  - purchased for \$1,500,
  - makes a single interest payment of \$100,
  - sold for \$1,600.
- What is the interest rate, rate of capital gain, rate of return?
- Suppose instead the sale price is \$1,400. What is the interest rate, rate of capital gain, rate of return?

# Maturity, Volatility, and Return

- Long-term debt instruments have a high degree of interest rate risk.
- **interest rate risk:** changes in interest rates over the life of the debt instrument influence the secondary market price of the bond, influencing capital gains and therefore rate of return.
- Prices and returns for long-term bonds are *more volatile* than short-term bonds.
- Interest payments are therefore typically higher for long-term bonds.

# Auto Loans

## Auto Loan

- Suppose you borrow \$15,000 to purchase a car.
- Six years of monthly payments at 4% APR
- What is the monthly payment on the loan?

## No Payments for First Three Months!

- Suppose you borrow \$15,000 to purchase a car.
- Six years of monthly payments at 4% APR
- No payments for first three months!
- What is the monthly payment on the loan?

# Coupon Bonds on the Secondary Market

- Suppose there is a coupon bond on the secondary market with face value of \$1,000, makes two payments per year, has annual coupon rate of 3.5%, and has 6 years until maturity.
- Suppose the current interest rate is 4%. Compute the semi-annual interest rate and the present value of the bond.
- Suppose you purchase the bond at a price equal to the present value. Suppose you hold it for 1.5 years and then sell it again on the secondary market. Suppose in 1.5 years, the interest rate is 3%. What will the present value of the bond be at that sell date?

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