ECO 305: Intermediate Macroeconomics
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Homework: One-Period Consumption / Leisure Model
Fall 2011

1. Suppose a consumer values consumption and leisure according to the utility function, $u(c, l) = c^{\frac{1}{2}} + ln(l)$.

   (a) Are the marginal utilities for consumption and leisure always positive or always negative? Demonstrate mathematically.

   (b) Does this utility function exhibit diminishing marginal utility for both consumption and leisure? Demonstrate mathematically.

2. Suppose a production function is given by $y = zN_d^{\frac{2}{3}} K^{\frac{1}{3}}$.

   (a) Does this production function exhibit diminishing marginal product of labor? Demonstrate mathematically.

   (b) Does this production function exhibit diminishing marginal product of capital? Demonstrate mathematically.

3. Suppose the government imposes a labor income tax, instead of a lump-sum tax. Let $\tau \in [0, 1)$ the proportion of labor income that is collected in taxes, so that the proportion of labor income a worker gets to keep is equal to $1 - \tau$. The consumer's real value of income is therefore equal to $(1 - \tau)w(h - l) + \pi$.

   (a) Illustrate the effects of the labor income tax on optimal labor supply and consumption demand choices compared to the situation in which there is no labor income tax. Assume that substitution effects outweigh income effects in the even these oppose each other.

   (b) Continue to assume a proportional labor income tax. Suppose the consumer enjoys consumption and leisure according to the utility function, $u(c, l) = c^{\frac{4}{5}} l^{\frac{1}{5}}$. Find equations for the optimal consumption and leisure choices. Mathematically demonstrate the effect increasing taxes has on these optimal choices.