

# Economic Growth: Solow Growth Model

ECO 305: Intermediate Macroeconomics

# Goals

1 / 20

Use Solow growth model theory to explain...

- 1 Why some countries have high rates of growth and other have low rates of growth
- 2 What factors affect economic development and growth
- 3 Shortcomings of the theory

## Reading and Exercises

2 / 20

- **Williamson, Chapter 7, pp. 249-255: Solow Model**
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- **Canvas Quiz due Wednesday 11:59 PM.**  
Multiple-choice, 10 questions, unlimited attempts allowed, only best score counts
- **Homework/Exercise due Friday 11:59 PM.** We will work together in class on Thursday

## Reading and Exercises

2 / 20

- Williamson, Chapter 7, pp. 249-255: Solow Model
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- **Canvas Quiz due Wednesday 11:59 PM.**  
Multiple-choice, 10 questions, unlimited attempts allowed, only best score counts
- **Homework/Exercise due Friday 11:59 PM.** We will work together in class on Thursday

## Reading and Exercises

2 / 20

- Williamson, Chapter 7, pp. 249-255: Solow Model
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- **Canvas Quiz due Wednesday 11:59 PM.**  
Multiple-choice, 10 questions, unlimited attempts allowed, only best score counts
- **Homework/Exercise due Friday 11:59 PM.** We will work together in class on Thursday

## Reading and Exercises

2 / 20

- Williamson, Chapter 7, pp. 249-255: Solow Model
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- **Canvas Quiz due Wednesday 11:59 PM.**  
Multiple-choice, 10 questions, unlimited attempts allowed, only best score counts
- **Homework/Exercise due Friday 11:59 PM.** We will work together in class on Thursday

# Economic Growth Facts Over Time

3 / 20

- Before the industrial revolution in about 1800, standards of living did not grow much over time.
- Since the industrial revolution, per-capita income growth has grown slowly and steadily in the richest countries
  - The average growth rate of per-capita income in the richest countries is about 1-3%.

# Economic Growth Facts Over Time

3 / 20

- Before the industrial revolution in about 1800, standards of living did not grow much over time.
- Since the industrial revolution, per-capita income growth has grown slowly and steadily in the richest countries
  - The average growth rate of per-capita income in the richest countries is about 1-3%.



# Economic Growth Facts Over Time

3 / 20

- Before the industrial revolution in about 1800, standards of living did not grow much over time.
- Since the industrial revolution, per-capita income growth has grown slowly and steadily in the richest countries
  - The average growth rate of per-capita income in the richest countries is about 1-3%.

# Economic Growth Facts Across Countries

4 / 20

- Before the industrial revolution, standards of living were similar across much of the world.
- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

# Economic Growth Facts Across Countries

4 / 20

- Before the industrial revolution, standards of living were similar across much of the world.
- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

# Economic Growth Facts Across Countries

4 / 20

- Before the industrial revolution, standards of living were similar across much of the world.
- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

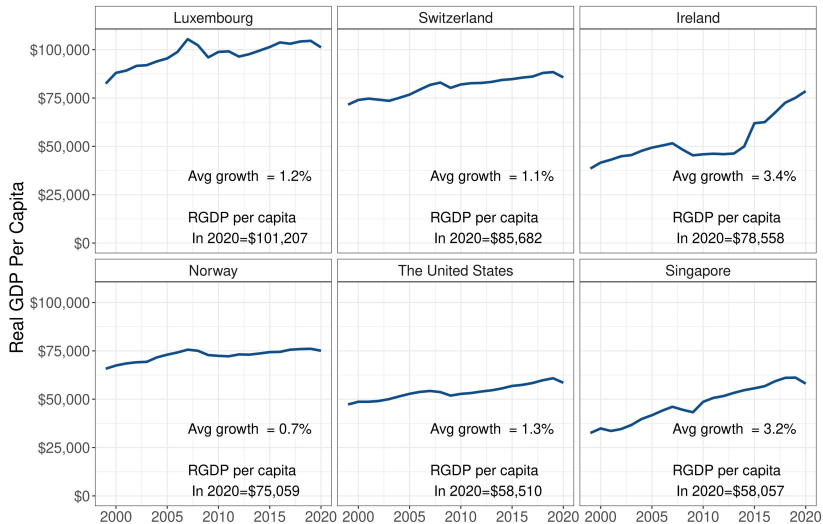
# Economic Growth Facts Across Countries

4 / 20

- Before the industrial revolution, standards of living were similar across much of the world.
- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

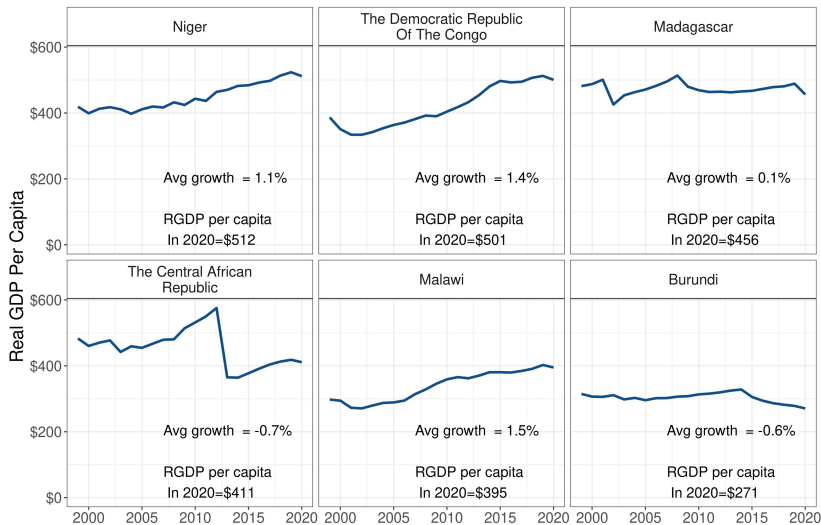
## Richest Economies (Real GDP Per Capita in 2020)

5 / 20



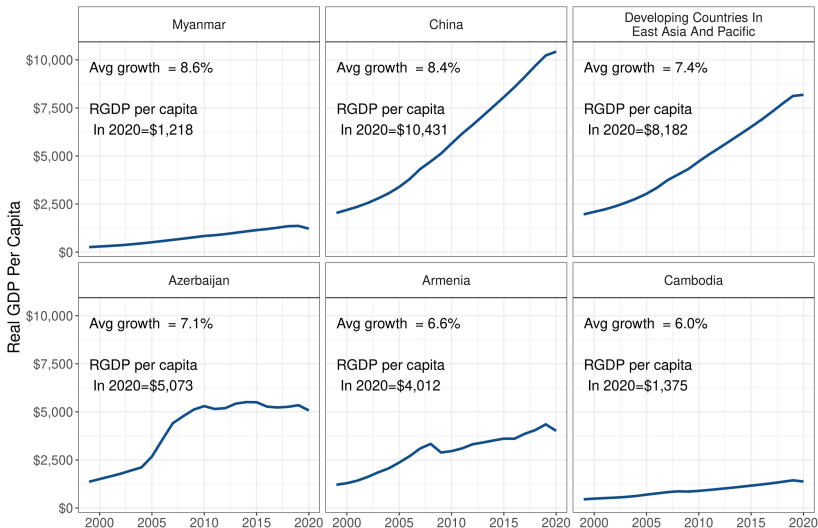
## Poorest Economies (Real GDP Per Capita in 2020)

6 / 20



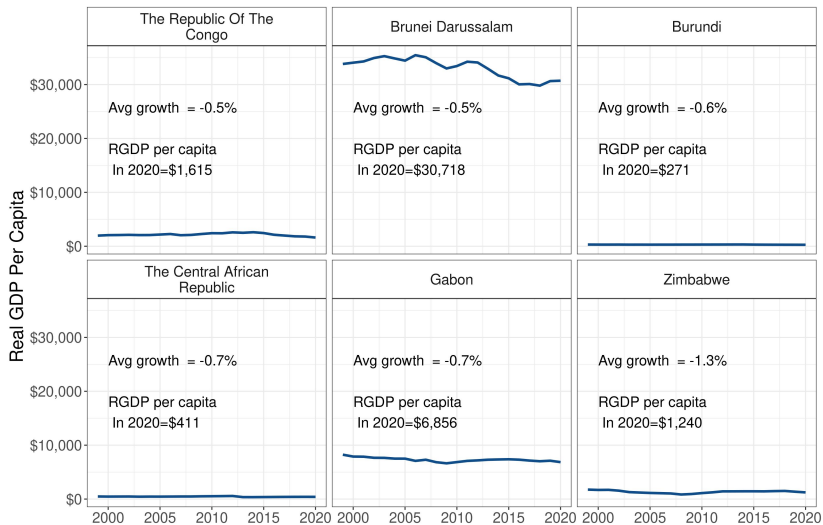
## Fastest Growing Economies (1999-2019)

7 / 20





# Slowest Growing Economies (1999-2019)



# Growth Covariates

9 / 20

- There is a negative relationship between population growth rates and per-capita income growth rates.
- There is a positive relationship between investment rate (as a percentage of real GDP) and per-capita income for **lesser developed countries**.
- There is a negative relationship between investment rate and per-capita income for **more developed countries**.

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY'$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY'$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY'$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker



# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker



# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

# Foundations of Solow Growth Model

10 / 20

## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
- $n \in (0, 1)$ : exogenous population growth rate

## Government Behavior

- Government budget:  $G = T + B$
- Government borrowing:  $B = bY$
- $b \in (0, 1)$  is exogenous rate of government borrowing
- Let  $s_N = s - b$ ,  $s_N \in (0, 1)$  denote national savings rate

## Investment and Capital

- Evolution of Capital Stock:  
 $K' = (1 - d)K + I$
- $K$  and  $K'$ : capital today / future
- $I$ : real quantity of investment
- $d \in (0, 1)$ : exogenous depreciation rate

## Production Function

- Production function:  $Y = zf(K, N)$
- Per-worker production function:  
 $y = zf(k)$
- $y$  is output/worker ( real GDP per capita)
- $k$  is existing quantity of capital stock per worker

## Further Assumptions

11 / 20

- Diminishing marginal of capital
- Constant returns to scale: When an economy increases all of its factors of production (i.e. both labor and capital) by the same percentage, production goes up by the same percentage

## Further Assumptions

11 / 20

- Diminishing marginal of capital
- Constant returns to scale: When an economy increases all of its factors of production (i.e. both labor and capital) by the same percentage, production goes up by the same percentage



# Results of the Solow Growth Model

## Conclusions

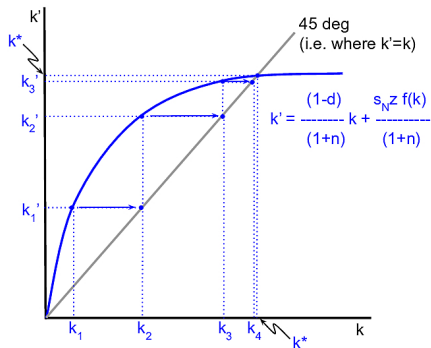
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

Savings and Investment:

- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12/ 20

## Conclusions

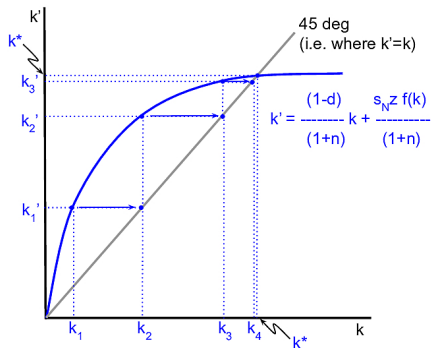
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

Savings and Investment:

- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12 / 20

## Conclusions

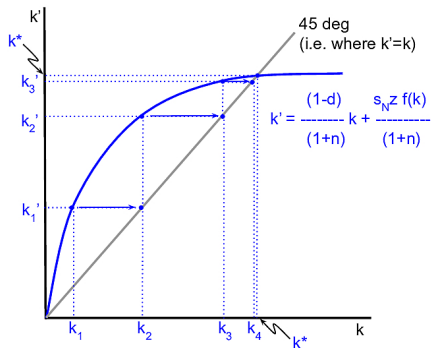
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

Savings and Investment:

- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12 / 20

## Conclusions

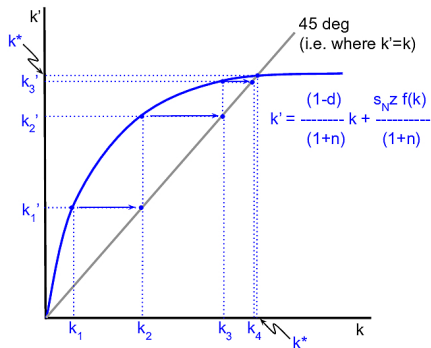
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N Z}{1+n} \right) f(k)$$

Savings and Investment:

- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12 / 20

## Conclusions

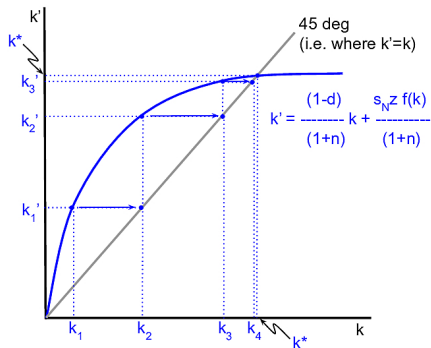
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

Savings and Investment:

- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12/ 20

## Conclusions

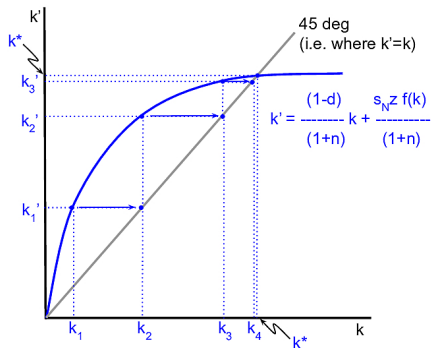
Evolution of capital stock per worker:

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

Savings and Investment:

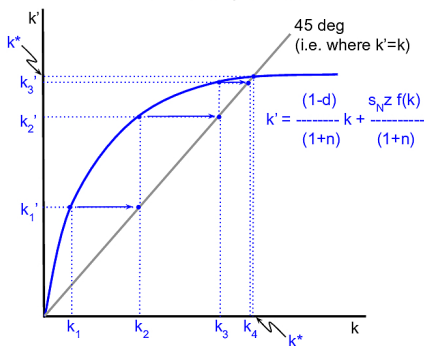
- $I = S - B$
- $I = sY - bY = (s - b)Y = s_N Y$
- Investment = National Savings

Solow Growth: Capital Evolution

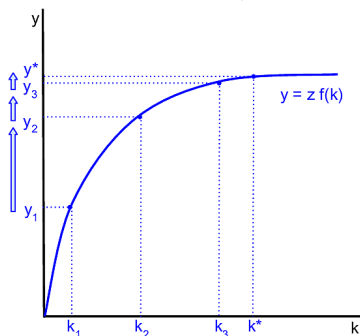


# Economic Growth in Solow Model

Solow Growth: Capital Evolution



Solow Growth: Output Evolution

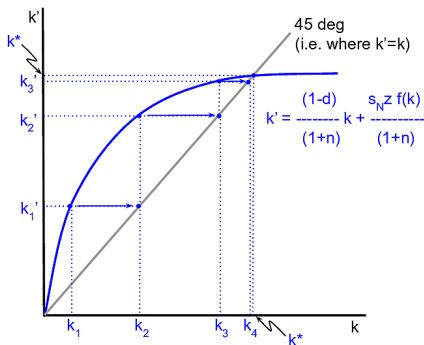


## Economic Growth

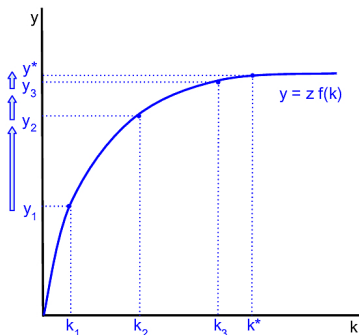
- Increases in real GDP per capita higher at lower levels of capital stock per worker / lower levels of real GDP per capita
- Due to diminishing marginal product of capital

# Economic Growth in Solow Model

Solow Growth: Capital Evolution



Solow Growth: Output Evolution



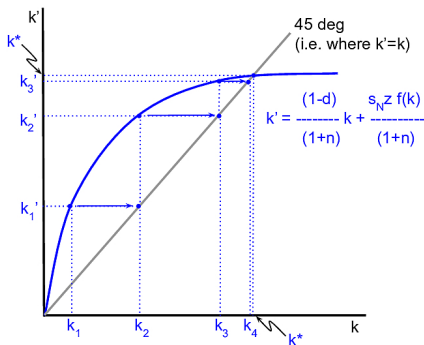
## Economic Growth

- Increases in real GDP per capita higher at lower levels of capital stock per worker / lower levels of real GDP per capita
- Due to diminishing marginal product of capital

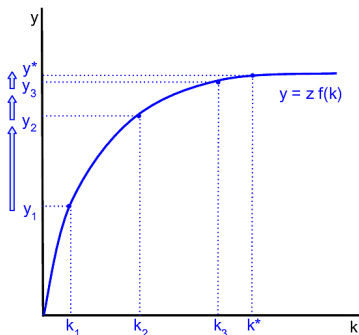


# Economic Growth in Solow Model

Solow Growth: Capital Evolution



Solow Growth: Output Evolution



## Economic Growth

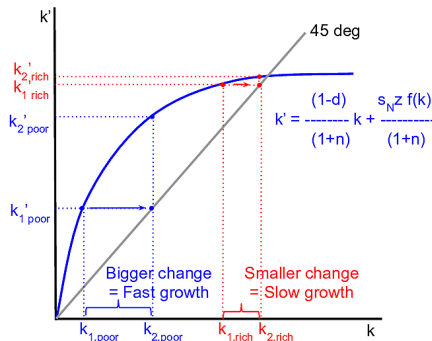
- Increases in real GDP per capita higher at lower levels of capital stock per worker / lower levels of real GDP per capita
- Due to diminishing marginal product of capital

# Lesser-Developed vs Developed Economies

## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution

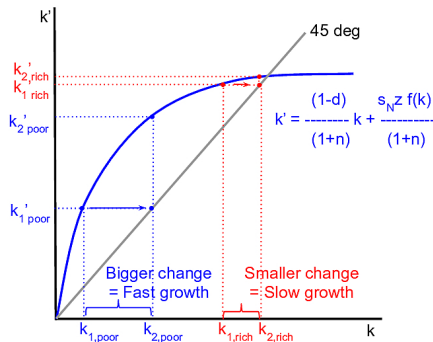


# Lesser-Developed vs Developed Economies

## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution

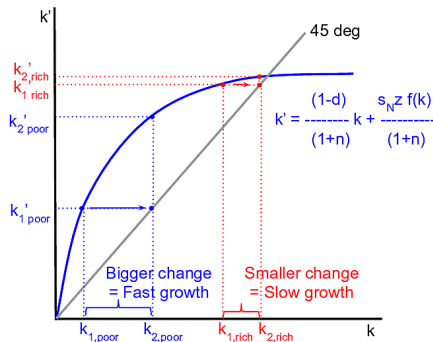


# Lesser-Developed vs Developed Economies

## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution

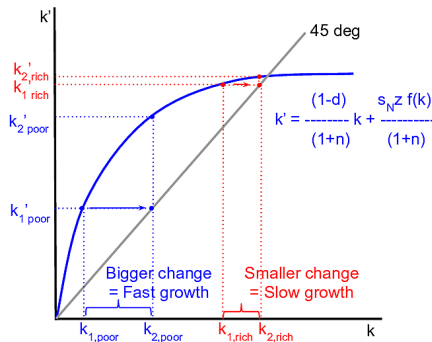


# Lesser-Developed vs Developed Economies

## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution

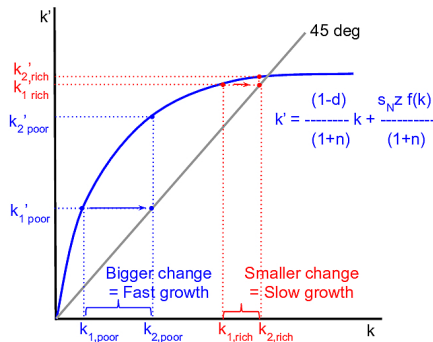


# Lesser-Developed vs Developed Economies

## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution



## Solving for Steady State

15 / 20

## Solving for Steady State

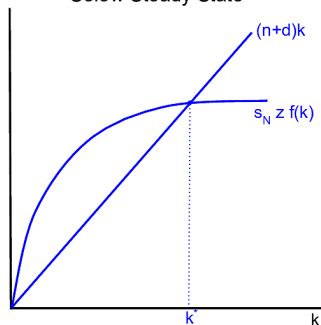
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

## Solow Steady State



## Factors Affecting Steady State:

- 1 z: Total factor productivity
- 2  $s_N$ : National savings rate

- 3 n: population growth rate
- 4 d: depreciation rate

# Solving for Steady State

15 / 20

## Solving for Steady State

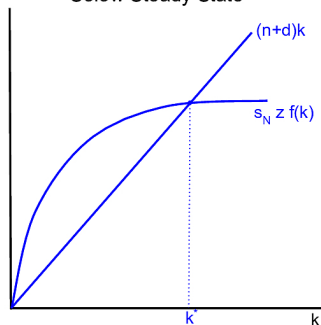
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

Solow Steady State



Factors Affecting Steady State:

- 1 z: Total factor productivity
- 2  $s_N$ : National savings rate

- 3 n: population growth rate
- 4 d: depreciation rate



# Solving for Steady State

15 / 20

## Solving for Steady State

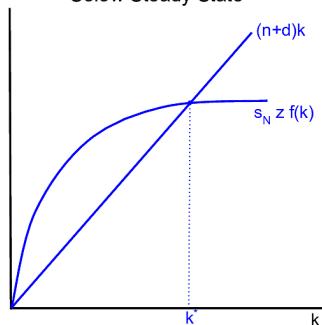
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

Solow Steady State



Factors Affecting Steady State:

- 1 z: Total factor productivity
- 2  $s_N$ : National savings rate

- 3 n: population growth rate
- 4 d: depreciation rate

# Solving for Steady State

15 / 20

## Solving for Steady State

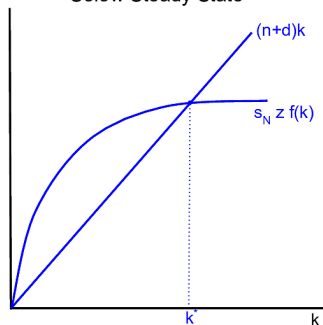
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

Solow Steady State



Factors Affecting Steady State:

- |                                   |                                |
|-----------------------------------|--------------------------------|
| ① $z$ : Total factor productivity | ③ $n$ : population growth rate |
| ② $s_N$ : National savings rate   | ④ $d$ : depreciation rate      |

# Solving for Steady State

15 / 20

## Solving for Steady State

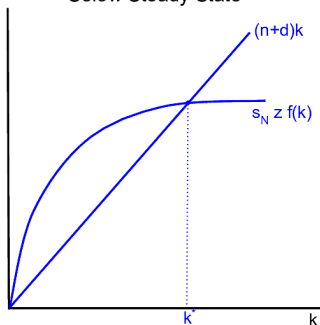
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

Solow Steady State



Factors Affecting Steady State:

- |                                   |                                |
|-----------------------------------|--------------------------------|
| ① $z$ : Total factor productivity | ③ $n$ : population growth rate |
| ② $s_N$ : National savings rate   | ④ $d$ : depreciation rate      |

# Solving for Steady State

15 / 20

## Solving for Steady State

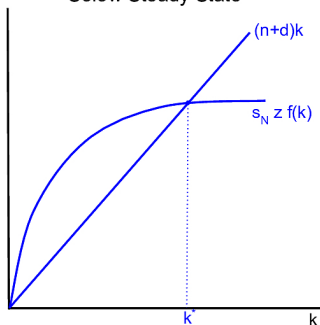
Use the following equation, and set  $k' = k = k^*$

$$k' = \left( \frac{1-d}{1+n} \right) k + \left( \frac{s_N z}{1+n} \right) f(k)$$

$$k^* = \left( \frac{1-d}{1+n} \right) k^* + \left( \frac{s_N z}{1+n} \right) f(k^*)$$

$$(n+d)k^* = s_N z f(k^*)$$

Solow Steady State



## Factors Affecting Steady State:

- |                                   |                                |
|-----------------------------------|--------------------------------|
| ① $z$ : Total factor productivity | ③ $n$ : population growth rate |
| ② $s_N$ : National savings rate   | ④ $d$ : depreciation rate      |

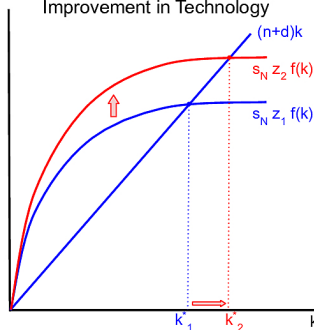
# Total Factor Productivity

## Improvement in Total Factor Productivity

- Increase in  $z$  causes production function and  $s_N z f(k^*)$  to pivot upward
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

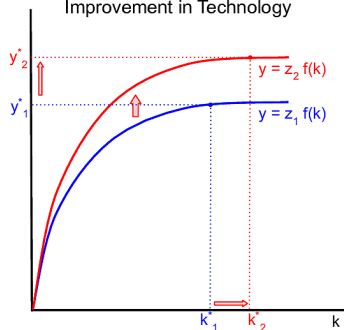
### Capital Per-Worker

Solow Growth:  
 Improvement in Technology



### Output Per-Worker

Solow Growth:  
 Improvement in Technology



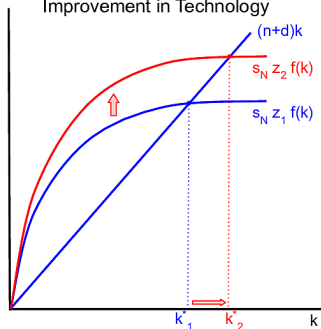
# Total Factor Productivity

## Improvement in Total Factor Productivity

- Increase in  $z$  causes production function and  $s_N z f(k^*)$  to pivot upward
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

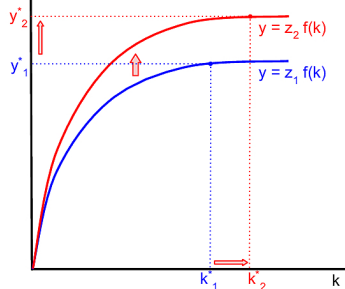
### Capital Per-Worker

Solow Growth:  
 Improvement in Technology



### Output Per-Worker

Solow Growth:  
 Improvement in Technology

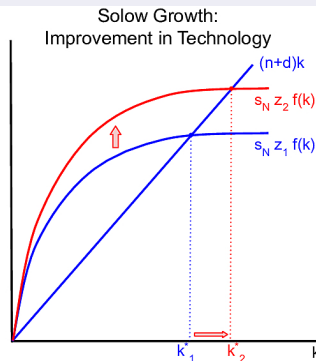


# Total Factor Productivity

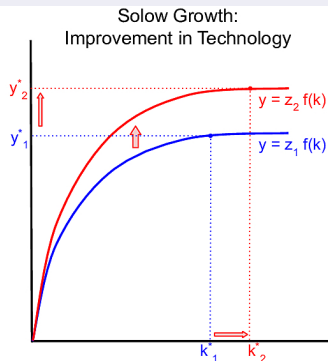
## Improvement in Total Factor Productivity

- Increase in  $z$  causes production function and  $s_N z f(k^*)$  to pivot upward
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

### Capital Per-Worker



### Output Per-Worker



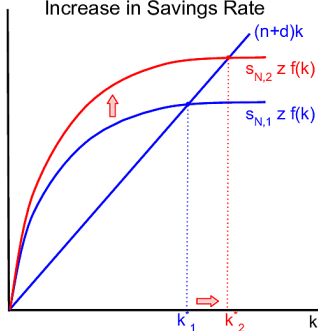
# National Savings Rate

## Increase in national savings rate

- Increase in  $s_N$  causes  $s_N z f(k^*)$  to pivot upward, but no change in production function
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

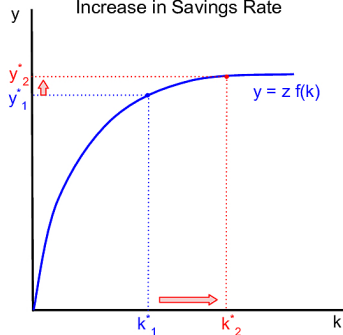
### Capital Per-Worker

Solow Growth:  
 Increase in Savings Rate



### Output Per-Worker

Solow Growth:  
 Increase in Savings Rate





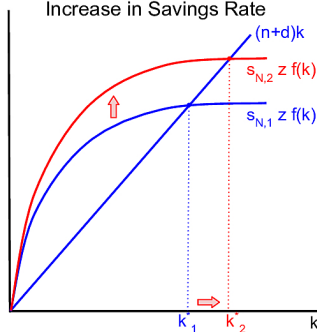
# National Savings Rate

## Increase in national savings rate

- Increase in  $s_N$  causes  $s_N z f(k^*)$  to pivot upward, but no change in production function
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

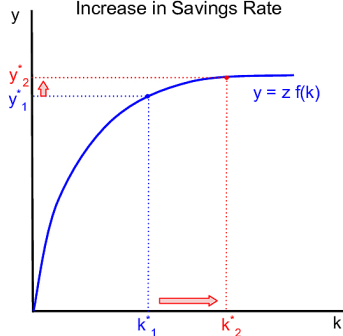
### Capital Per-Worker

Solow Growth:  
 Increase in Savings Rate



### Output Per-Worker

Solow Growth:  
 Increase in Savings Rate



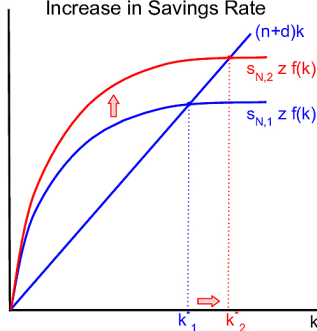
# National Savings Rate

## Increase in national savings rate

- Increase in  $s_N$  causes  $s_N z f(k^*)$  to pivot upward, but no change in production function
- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

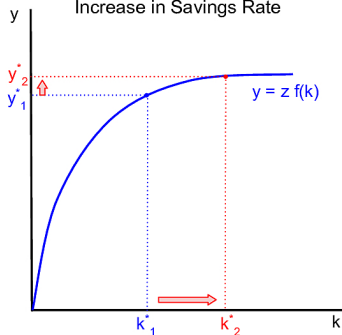
### Capital Per-Worker

Solow Growth:  
 Increase in Savings Rate



### Output Per-Worker

Solow Growth:  
 Increase in Savings Rate



# Population Growth Rate

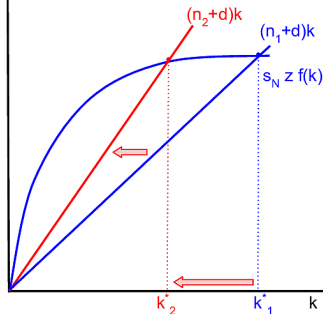
18 / 20

## Increase in Population Growth Rate

- Increase in  $n$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

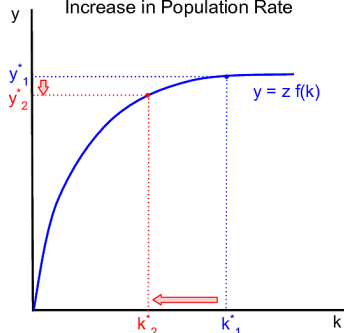
### Capital Per-Worker

Solow Growth:  
 Increase in Population Rate



### Output Per-Worker

Solow Growth:  
 Increase in Population Rate

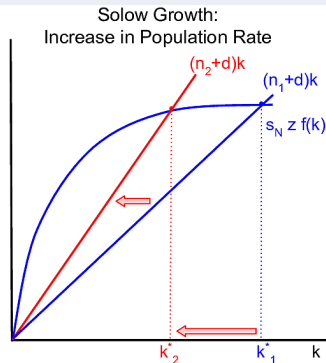


# Population Growth Rate

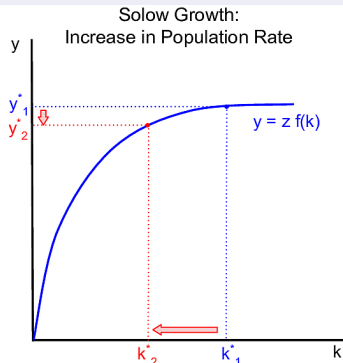
## Increase in Population Growth Rate

- Increase in  $n$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

### Capital Per-Worker



### Output Per-Worker

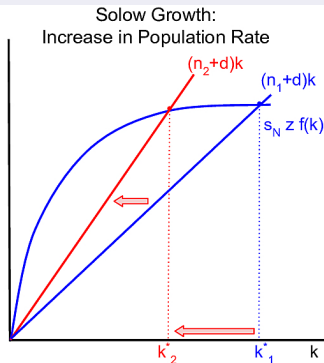


# Population Growth Rate

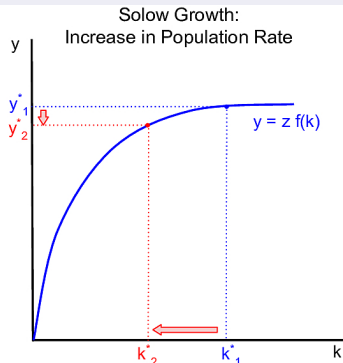
## Increase in Population Growth Rate

- Increase in  $n$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

### Capital Per-Worker



### Output Per-Worker



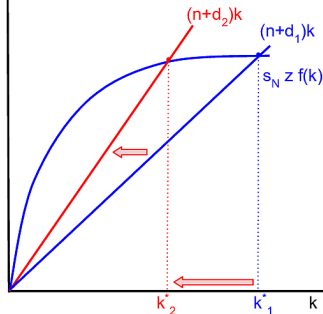
# Depreciation Rate

## Increase in Depreciation Rate of Capital

- Increase in  $d$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

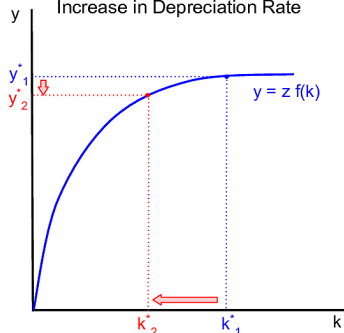
### Capital Per-Worker

Solow Growth:  
 Increase in Depreciation Rate



### Output Per-Worker

Solow Growth:  
 Increase in Depreciation Rate



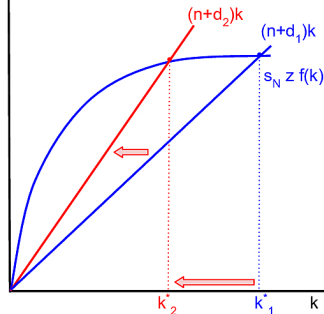
# Depreciation Rate

## Increase in Depreciation Rate of Capital

- Increase in  $d$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

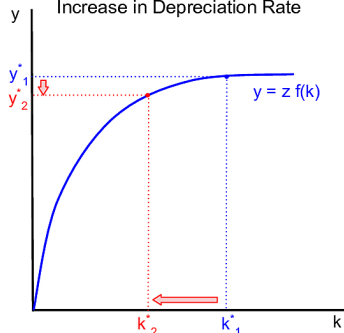
### Capital Per-Worker

Solow Growth:  
 Increase in Depreciation Rate



### Output Per-Worker

Solow Growth:  
 Increase in Depreciation Rate



# Depreciation Rate

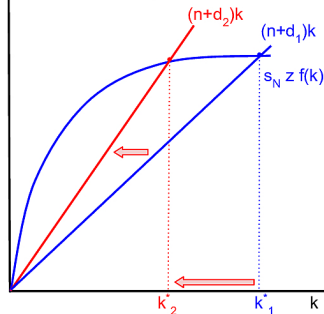
19 / 20

## Increase in Depreciation Rate of Capital

- Increase in  $d$  causes  $(n + d)k$  line to pivot upward
- Result: Lower  $k^*$  and  $y^*$ , i.e. lower steady state level of capital per worker, lower real GDP per capita

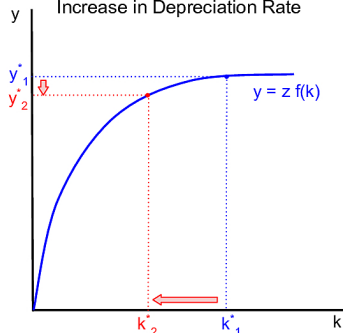
### Capital Per-Worker

Solow Growth:  
 Increase in Depreciation Rate



### Output Per-Worker

Solow Growth:  
 Increase in Depreciation Rate





# Model Shortcomings

20 / 20

- Not all lesser-developed countries have high rates of growth
- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth

# Model Shortcomings

20 / 20

- Not all lesser-developed countries have high rates of growth
- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth

# Model Shortcomings

20 / 20

- Not all lesser-developed countries have high rates of growth
- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth

# Model Shortcomings

20 / 20

- Not all lesser-developed countries have high rates of growth
- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth