# Dynamic Model with Money

ECO 305: Intermediate Macroeconomics

Fisher Equation
Market for Credit Goods
Market for Money
Dynamic General Equilibrium Model

**Goals**
Reading and Exercises

Goals                                                                1/ 21

- Introduce, money, price level, and inflation to the dynamic general equilibrium model
- Describe the relationship between inflation, real interest rates, and nominal interest rates
- Construct a microfounded model of money demand
- Make predictions for changes in quantity of money and price level in the dynamic general equilibrium model

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Goals
Reading and Exercises

## Reading and Exercises                                    2/ 21

- Dynamic model of money supply and demand Williamson, Chapter 12, pp. 442-457
- Complete dynamic macroeconomic model with money Williamson, Chapter 12, pp. 457-459
- Money neutrality Williamson, Chapter 12, pp. 460-464
- Shifts in Money Demand Williamson, Chapter 12, pp. 464-467
- **Canvas Quiz due Wed 11:59 PM**.
  Multiple-choice, 15 questions, unlimited attempts allowed, only best score counts
- **Homework/Exercise due Fri 11:59 PM**. We will work together in class on Thursday

## Notation

- **Fisher equation:** Relationship between real interest rate, nominal interest rate, and inflation rate
- **Price level notation:** $P$: price level today, $P'$: price level in future period
- **Inflation rate:** Denoted by $\pi$, growth rate of aggregate price level. $1 + \pi = \frac{P'}{P}$
- **Nominal interest rate (i):** Unadjusted rate of interest, not adjusted for changes in price level
- **Real interest rate (r):** Real rate, adjusted for inflation, paid for borrowed funds or earned from saving
- **Nominal bond:** Purchased in current period for \$1 by a lender. Seller/borrower pays \$(1+i) in future period.

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## Fisher Equation                                              4/ 21

**Nominal bond:** Purchased in current period for \$1 by a lender. Seller/borrower pays $\$(1+i)$ in future period.

$$\text{Real opportunity cost of buying bond} = \frac{\$1}{P}$$

$$\text{Real benefit to buying bond} = \frac{\$(1+i)}{P'}$$

$$\text{Gross rate of return to buying bond} = 1 + r = \frac{\text{Real benefit}}{\text{Real opportunity cost}}$$

$$1 + r = \frac{\dfrac{1+i}{P'}}{\dfrac{1}{P}} = \frac{1+i}{1+\pi}$$

## Fisher Equation

$$\text{Exact: } 1 + r = \frac{1+i}{1+\pi}$$

$$\text{Approximate: } r \approx i - \pi$$

- Real interest rate depends positively on nominal interest rate
- Inflation rate and real interest rates are negatively related
- An increase in expected inflation reduces the expected real return on a bond

Fisher Equation
**Market for Credit Goods**
Market for Money
Dynamic General Equilibrium Model

Credit vs Cash Goods Model
Shifts in Market for Credit Goods

# Credit vs Cash Goods

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## Bank Function

- Bank function: Facilitating transactions using credit services
- **Credit service** used here implies not needing cash at time of purchase

## Cash vs. Credit Goods

- Two types of goods: **cash goods** and **credit goods**
- Can think of as literally different goods (cash goods require cash or check)
- Normal goods, imperfectly substitutable
- Can think of as a choice to use credit services versus cash
- Real, per unit cost of using credit services: $q \in (0, 1)$
- Opportunity cost of cash good = nominal interest rate = $i \in (0, 1)$

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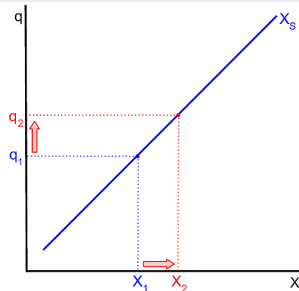
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# Supply of Credit Goods

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## Supply of Credit Services

- Banks provide credit services; quantity $= X$
- Price received for credit services $= q$
- Assume diminishing returns / increasing marginal costs
- Operating costs: technology and communication networks, research customers' credit histories, customer assistance, etc.
- Upward sloping supply curve

## Supply Curve

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Shifts in Market for Credit Goods

# Utility Maximization for Credit vs Cash Goods    8/ 21

## Utility Max Framework

- Two goods: Cash goods $= A$, Credit goods $= X$
- Nominal price of cash good (including opportunity cost) $= P(1+i)$
- Nominal price of credit good $= P(1+q)$

## Utility Maximization



Credit vs Cash Goods

A (Cash Goods)

Slope of Indifference Curve $= - \dfrac{MU_X}{MU_A}$

Slope of budget line $= - \dfrac{1+q}{1+i}$

$A_1$

$u_1$

$X_1$

X (Credit goods)

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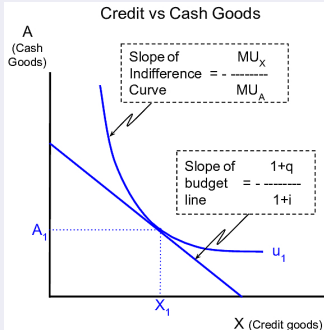
# Demand for Credit Goods

Suppose there is an **increase in price of credit services (q)**
(Focusing on substitution effects)



Utility Maximization

Credit vs Cash Goods



Utility Maximization

Fisher Equation
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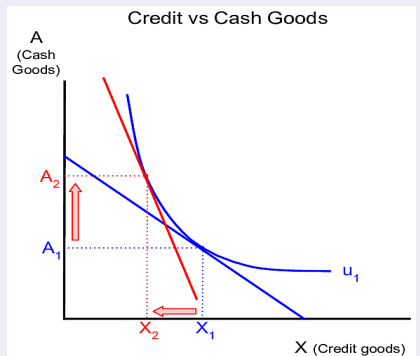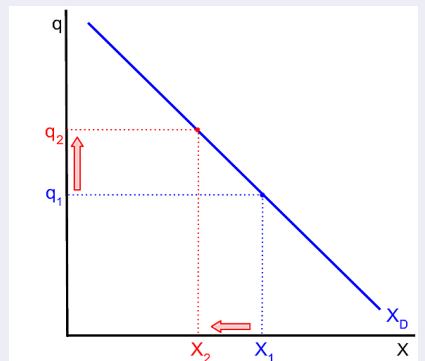
## Equilibrium In Credit Services Market

### Demand and Supply

- Demand depends on utility maximization
- Demand shifts with changes in interest rate and income
- Supply depends on profit maximization of banking sector
- Supply shifts with cost of providing credit services

### Equilibrium

Fisher Equation
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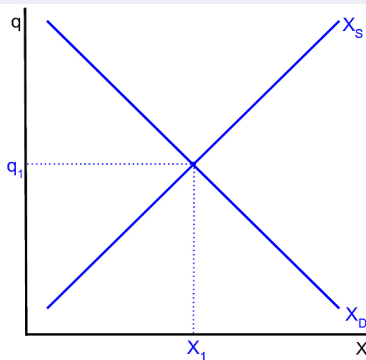
## Shift in Demand for Credit Goods: Interest Rate    11/ 21

Suppose there is an **increase in the interest rate (i)**
(Focusing on substitution effects)



Utility Maximization



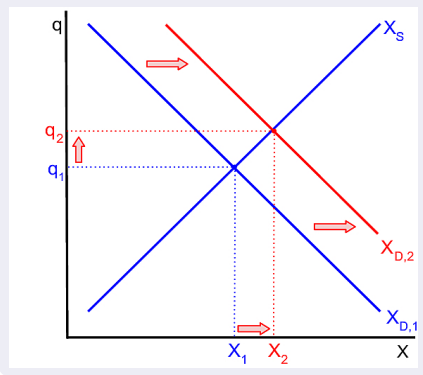Utility Maximization

Fisher Equation
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## Shift in Demand for Credit Goods: Income

Suppose there is an **increase in income**

Fisher Equation
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Credit vs Cash Goods Model
**Shifts in Market for Credit Goods**

# Shift in Supply of Credit Services

Suppose there is an improvement in financial technology, lowering the costs for banks to provide financial services
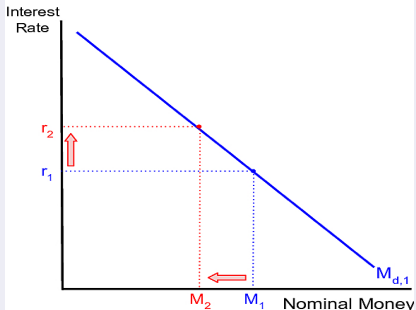


Utility Maximization



Utility Maximization

Fisher Equation
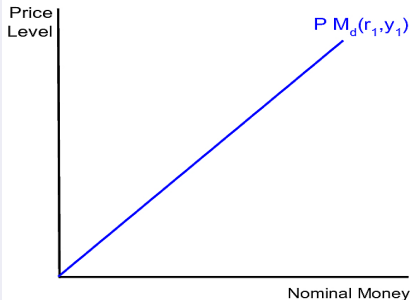Market for Credit Goods
**Market for Money**
Dynamic General Equilibrium Model

**Money Market Equilibrium**
Shifts in Money Demand
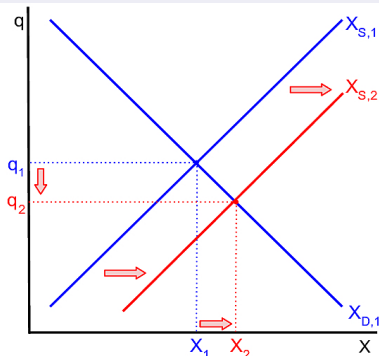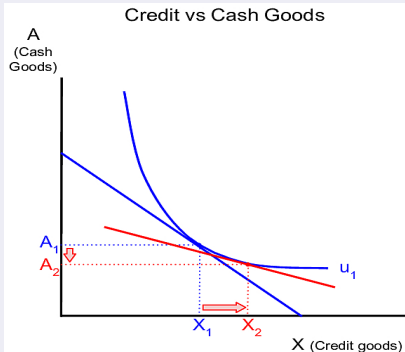
# Money Demand

Real money demand is the equilibrium outcome for cash goods, $A$.

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**Money Market Equilibrium**
Shifts in Money Demand

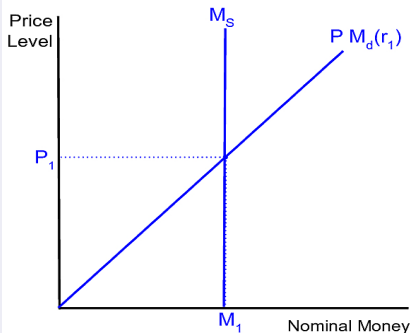# Money Market Equilibrium

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## Money Supply & Equilibrium

- Money supply is vertical where the central bank decides it should be
- Equilibrium price level is where money demand equals money supply
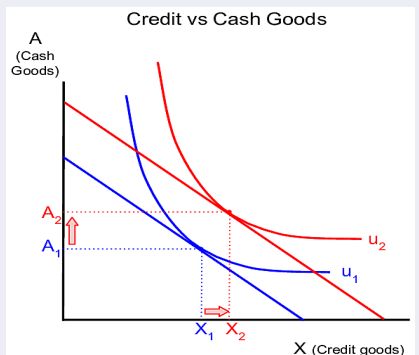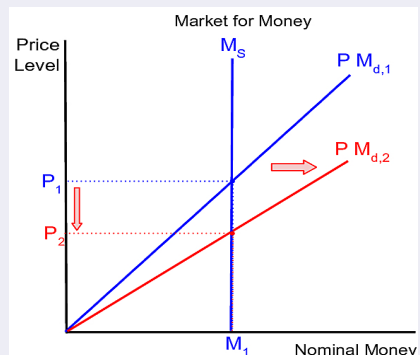
## Equilibrium

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Money Market Equilibrium
**Shifts in Money Demand**

## Shift in Demand for Money: Income

Suppose there is an **increase in income**



### Utility Maximization

Credit vs Cash Goods

A (Cash Goods) — $A_2$, $A_1$

$u_2$

$u_1$

$X_1$ $X_2$

X (Credit goods)

### Market for Money

Market for Money

Price Level

$M_S$

$P\,M_{d,1}$

$P\,M_{d,2}$

$P_1$

$P_2$

$M_1$

Nominal Money

Fisher Equation
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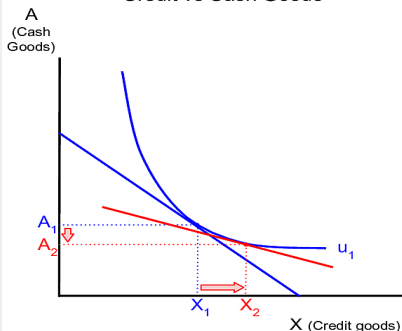Money Market Equilibrium
**Shifts in Money Demand**

# Shift in Demand for Money: Interest Rate          17/ 21

Suppose there is an **increase in interest rates**

## Utility Maximization



Credit vs Cash Goods

A
(Cash
Goods)

$A_1$
$A_2$

$u_1$

$x_1$  $x_2$

X (Credit goods)

## Market for Money



Price
Level

$M_S$   $P\,M_d(r_2)$   $P\,M_d(r_1)$

$P_2$

$P_1$

$M_1$   Nominal Money

Fisher Equation
Market for Credit Goods
**Market for Money**
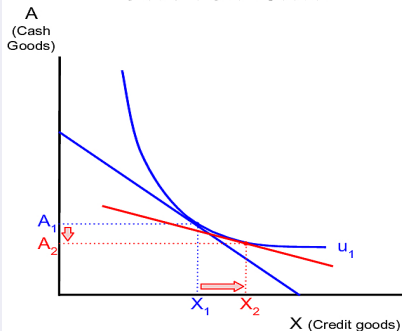Dynamic General Equilibrium Model

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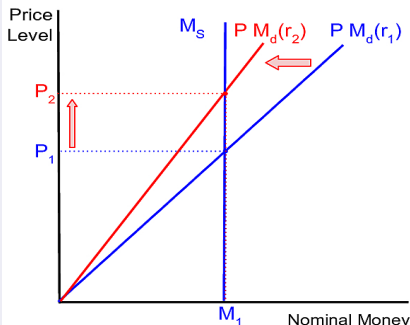
# Shift in Demand for Money: Financial Innovation      18/ 21

A financial innovation **decreases eqm price of credit services**
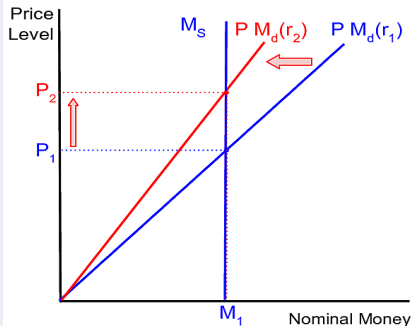
Fisher Equation
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**Combining Market for Money with DGE**
Money Surprises

## Combine with Dynamic General Equilibrium Model   19/ 21

### Dynamic General Equilibrium

- Solve the model: Improvement in productivity
- Solve the model: Consumer confidence falls (expectation of lower wages in future)
- Money neutrality: Federal Reserve increases money supply

### Strategy

- Start on outsides (supply side and demand side)
- Bring findings inward to output supply/demand
- Take results outward *and downward to money market*

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## Money Surpises                                                    20/ 21

- Suppose Fed increases money supply
- Price rises $+$ real wage stays same $\rightarrow$ nominal wage increased
- Suppose consumers were perceived increase in nominal wage was an increase in real wage
- Monetary policy has real effects, but is welfare reducing

## Reading and Exercises

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- Complete dynamic macroeconomic model with money Williamson, Chapter 12, pp. 457-459
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