

# Economic Growth: Solow Growth Model

ECO 305: Intermediate Macroeconomics

# Goals

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Use Solow growth model theory to explain...

- 1 Why some countries have high rates of growth and other have low rates of growth
- 2 What factors affect economic development and growth
- 3 Shortcomings of the theory

## Reading and Exercises

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- **Williamson, Chapter 7, pp. 249-255: Solow Model**
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- **Canvas Quiz due Wednesday 11:59 PM.**  
Multiple-choice, 15 questions, unlimited attempts allowed, only best score counts
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# Economic Growth Facts Over Time

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- Before the industrial revolution in about 1800, standards of living did not grow much over time.
- Since the industrial revolution, per-capita income growth has grown slowly and steadily in the richest countries
  - The average growth rate of per-capita income in the richest countries is about 1-3%.

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# Economic Growth Facts Across Countries

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- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

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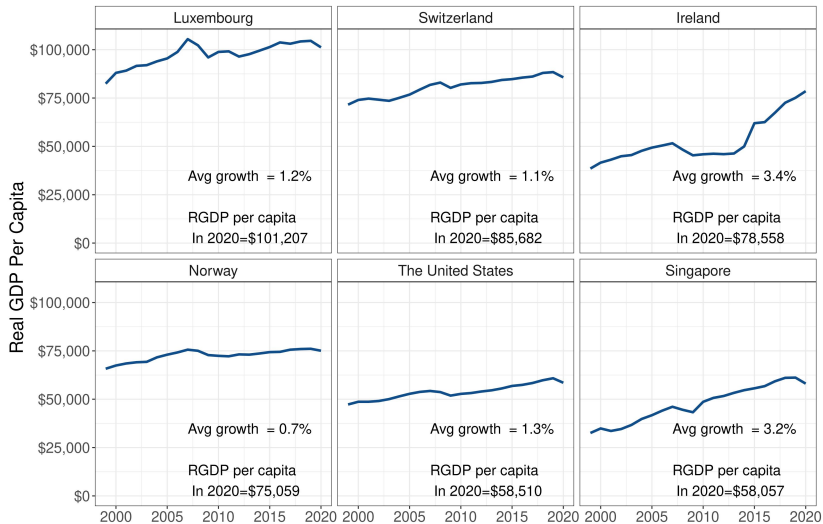
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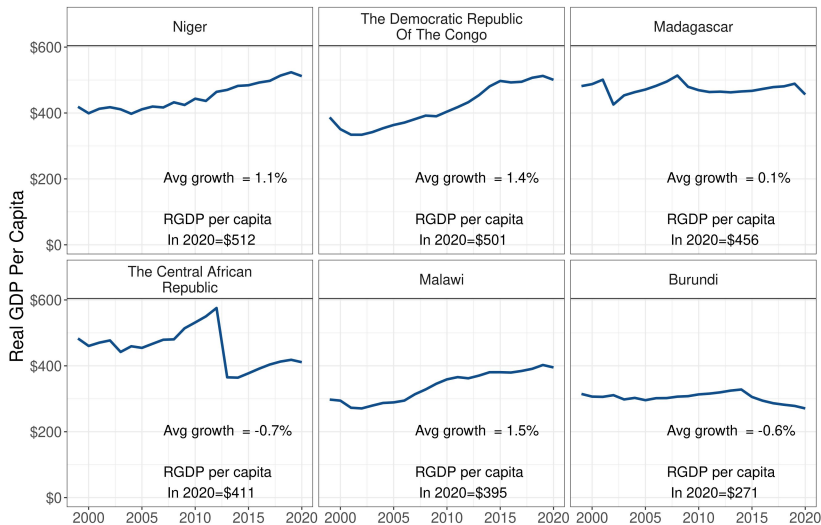
## Richest Economies (Real GDP Per Capita in 2020)

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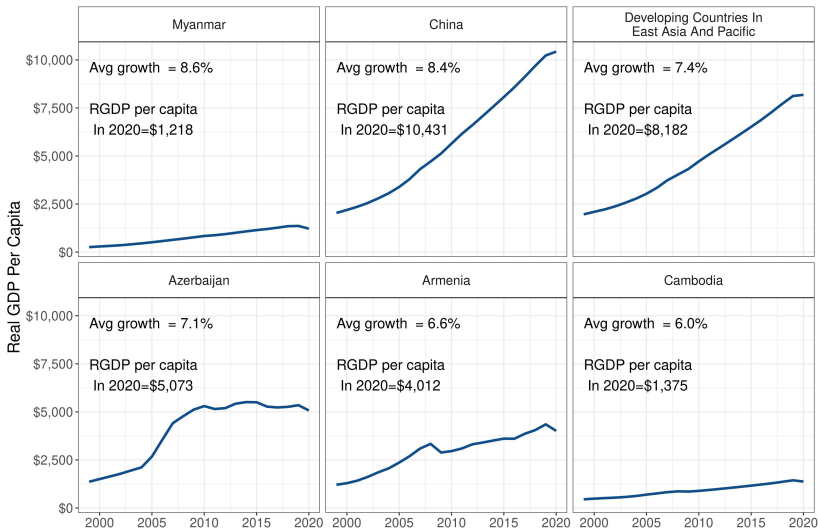
## Poorest Economies (Real GDP Per Capita in 2020)

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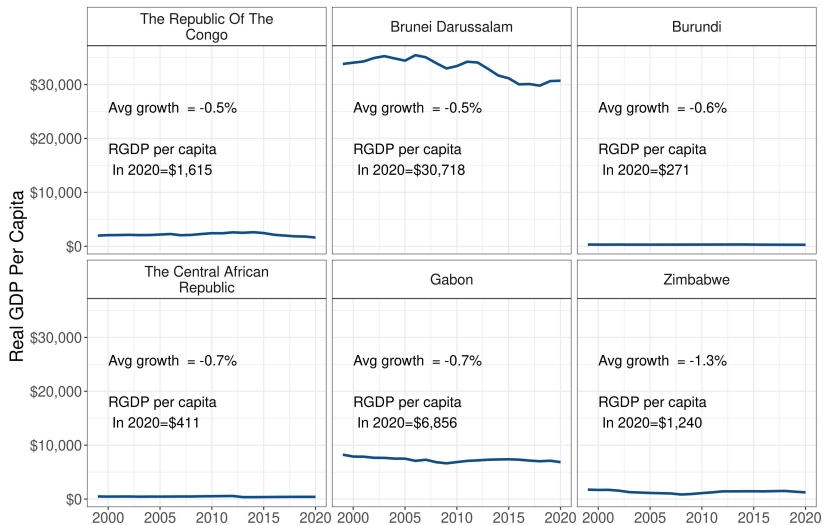
## Fastest Growing Economies (1999-2019)

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# Slowest Growing Economies (1999-2019)



# Growth Covariates

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- There is a negative relationship between population growth rates and per-capita income growth rates.
- There is a positive relationship between investment rate (as a percentage of real GDP) and per-capita income for **lesser developed countries**.
- There is a negative relationship between investment rate and per-capita income for **more developed countries**.

# Foundations of Solow Growth Model

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## Consumer Behavior

- Private consumer savings:  $S = sY$
- Population growth:  $N' = (1 + n)N$
- $s \in (0, 1)$ : exogenous savings rate
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## Government Behavior

- Government budget:  $G = T + B$
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## Further Assumptions

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- Diminishing marginal of capital
- Constant returns to scale: When an economy increases all of its factors of production (i.e. both labor and capital) by the same percentage, production goes up by the same percentage

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# Results of the Solow Growth Model

## Conclusions

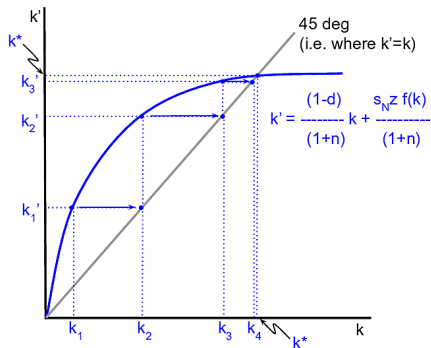
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Solow Growth: Capital Evolution



# Results of the Solow Growth Model

12/ 21

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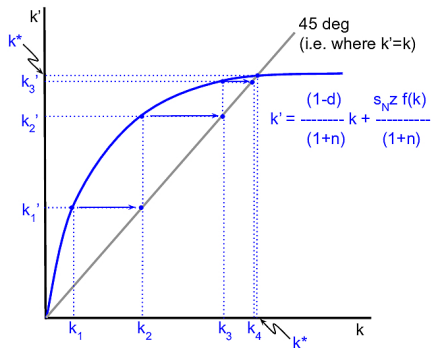
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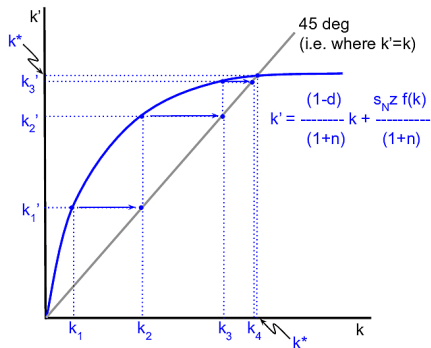
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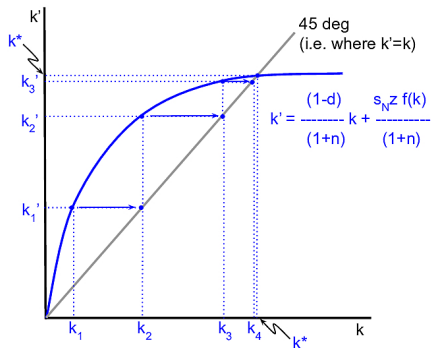
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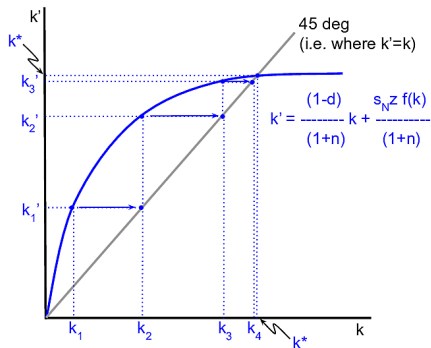
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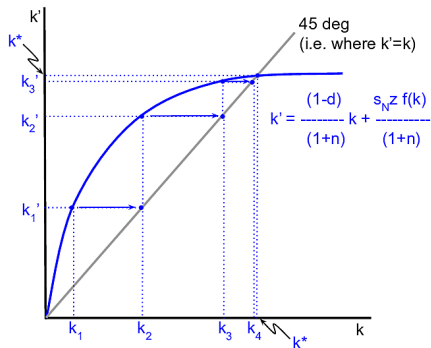
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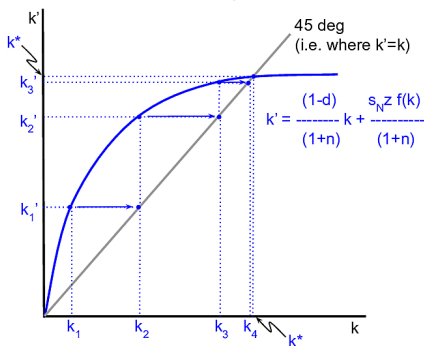
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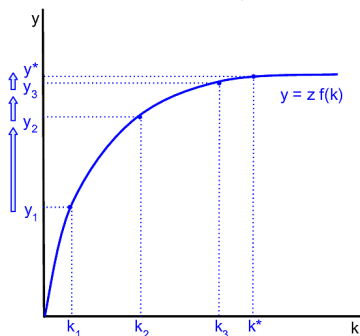
# Economic Growth in Solow Model

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Solow Growth: Capital Evolution



Solow Growth: Output Evolution

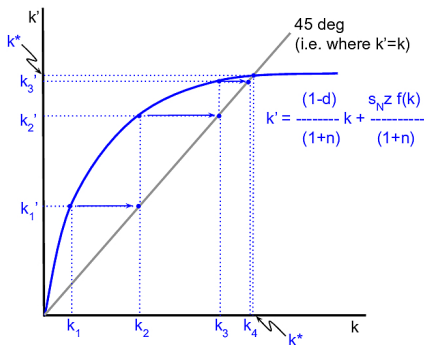


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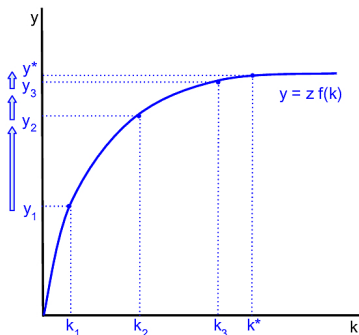
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Solow Growth: Capital Evolution



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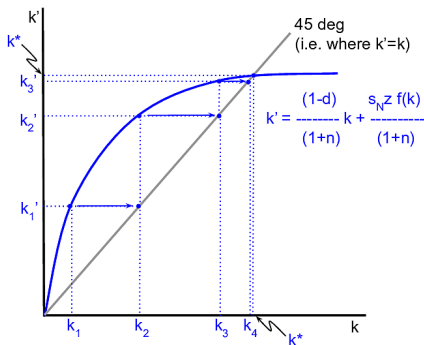
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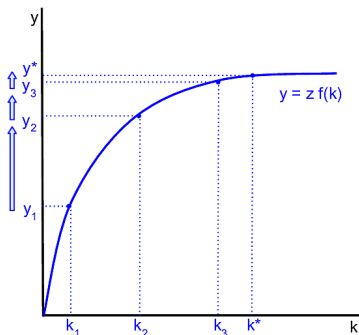


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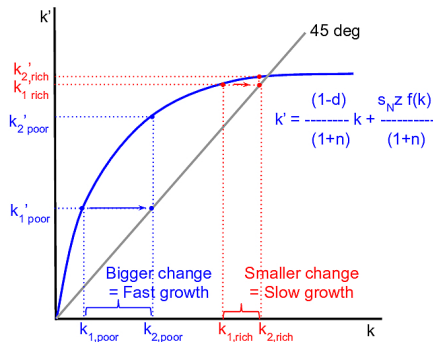
# Lesser-Developed vs Developed Economies

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## Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state ( $k^*, y^*$ ), only economic growth would come from shift in  $z$
- **Convergence:** Lesser-developed economies eventually catch up to highly-developed economies

Solow Growth: Capital Evolution

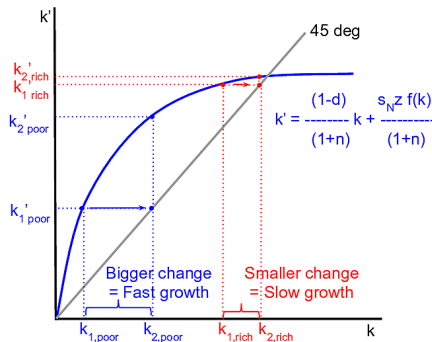


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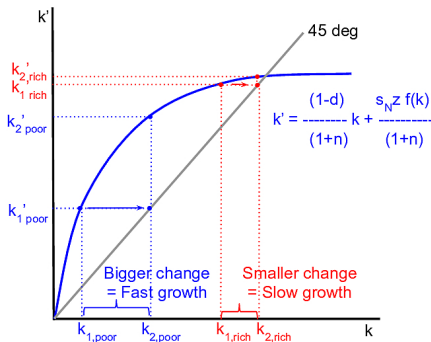


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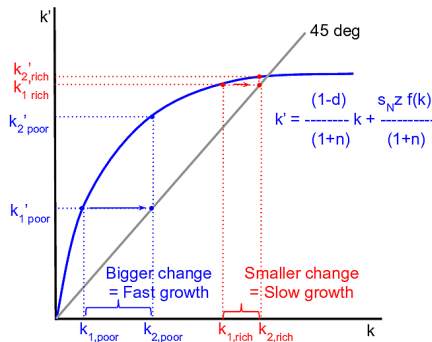


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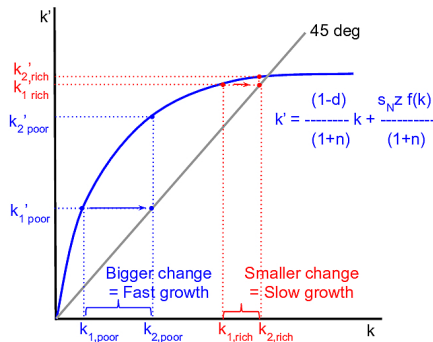
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# Solving for Steady State

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## Solving for Steady State

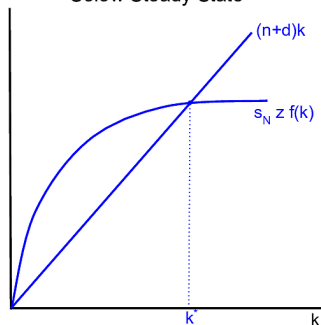
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Solow Steady State



Factors Affecting Steady State:

- 1 z: Total factor productivity
- 2  $s_N$ : National savings rate

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# Solving for Steady State

15 / 21

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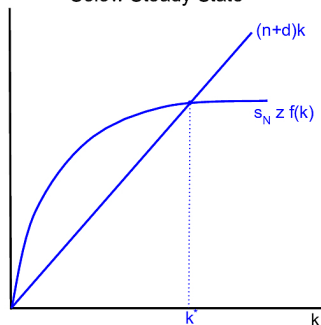
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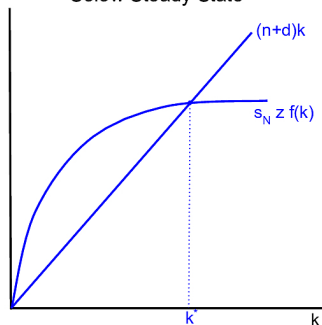
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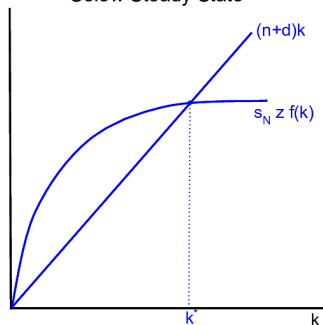
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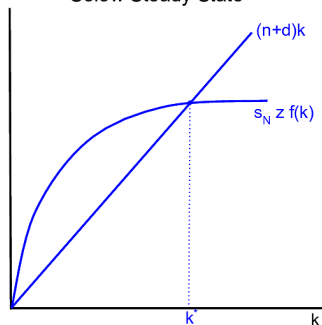
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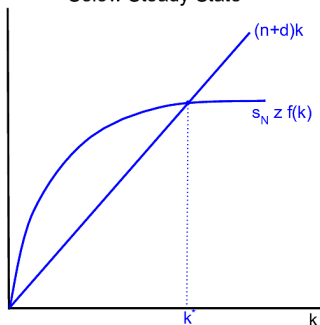
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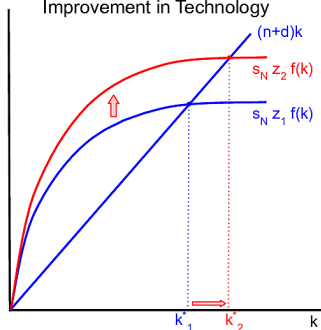
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## Improvement in Total Factor Productivity

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- Result: Higher  $k^*$  and  $y^*$ , i.e. higher steady state level of capital per worker, higher real GDP per capita

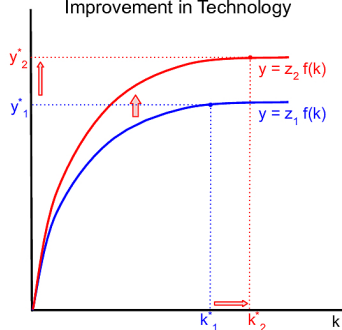
### Capital Per-Worker

Solow Growth:  
 Improvement in Technology



### Output Per-Worker

Solow Growth:  
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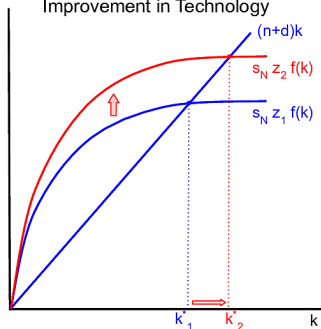
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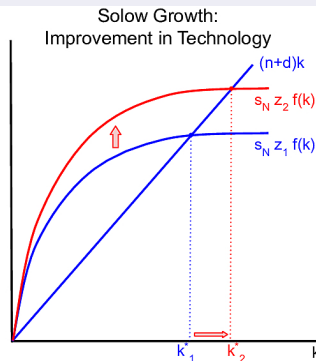


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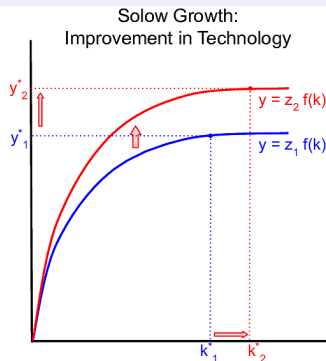
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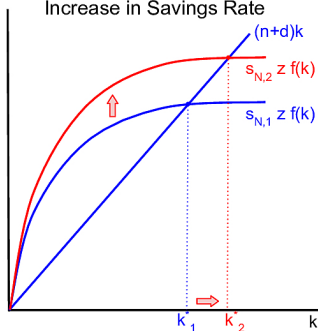
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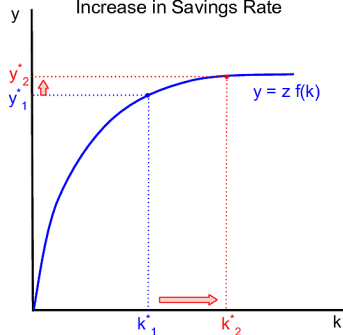
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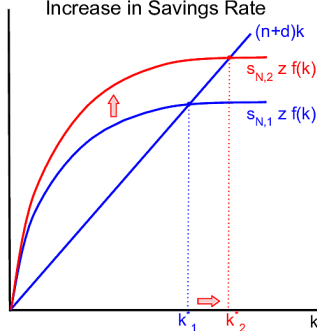
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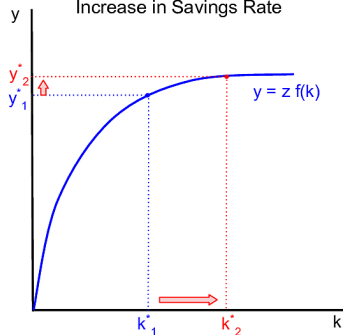
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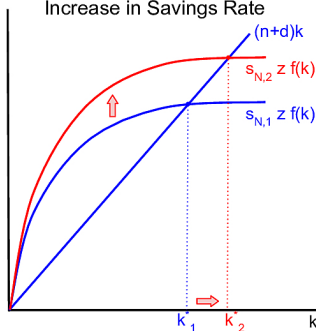
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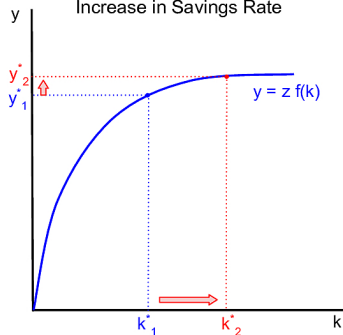
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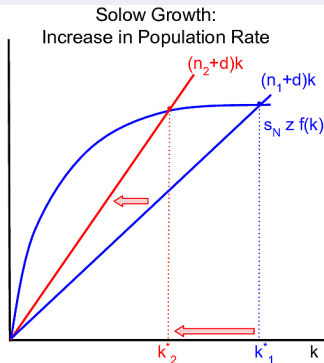
# Population Growth Rate

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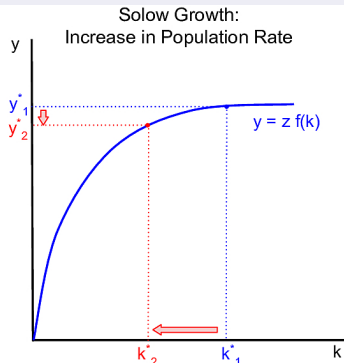
## Increase in Population Growth Rate

- Increase in  $n$  causes  $(n + d)k$  line to pivot upward
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### Capital Per-Worker



### Output Per-Worker



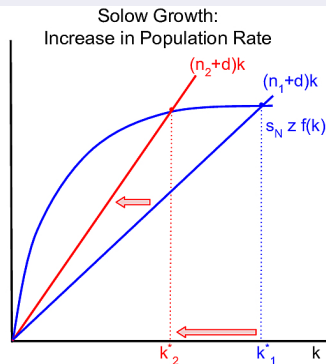
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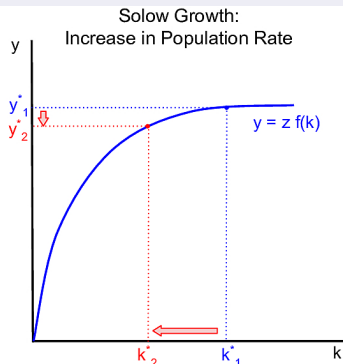
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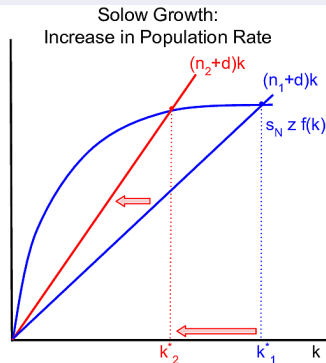
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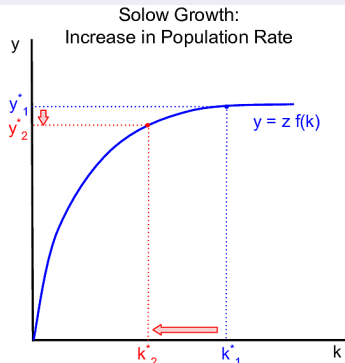
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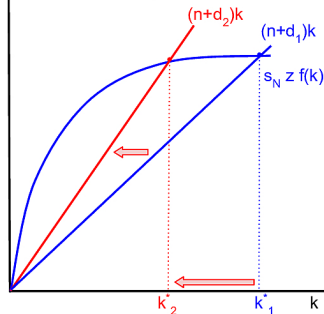
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## Increase in Depreciation Rate of Capital

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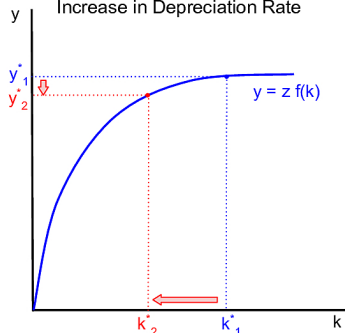
### Capital Per-Worker

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 Increase in Depreciation Rate



### Output Per-Worker

Solow Growth:  
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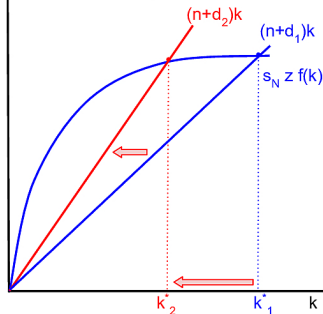
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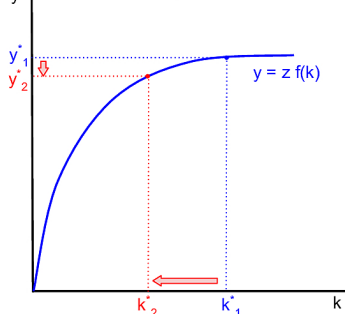
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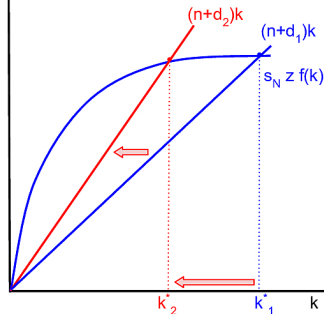
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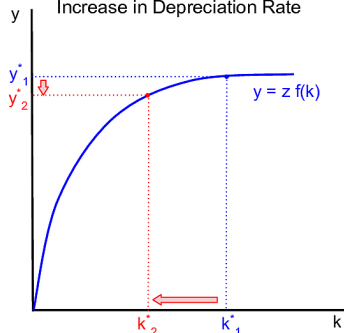
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### Output Per-Worker

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 Increase in Depreciation Rate





# Model Shortcomings

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- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth

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## Reading and Exercises

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- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
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