Analysis of Variance (ANOVA)

MGMT 662: Integrative Research Project

August 7, 2008.
Goals of this class meeting

- Learn how to test for significant differences in means from two or more groups.
- Learn how to account for an additional factor.
- Learn how to test for significant differences in medians from two or more groups. Why?
One-Way ANOVA

- Method for testing for significant differences among means from two or more groups.
- Essentially an extension of the t-test for testing the differences between two means.
- Uses measures of variance to measure for differences in means.
- Total variation in your data is decomposed into two components:
  - Among-group variation: variability that is due to differences among groups, also called explained variation.
  - Within-group variation: total variability within each of the groups, this is unexplained variation.
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**Variance Decomposition**

- **Sum of squares groups (SSG):**
  \[
  SSG = \sum_{k=1}^{K} n_k (\bar{x}_k - \bar{x})^2
  \]
  - \(K\): number of groups.
  - \(\bar{x}_k\) is the mean of group \(k\).
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- **Sum of squares within-group (SSW):**
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**Hypothesis Test**

- **Null hypothesis:** \( \mu_1 = \mu_2 = \ldots = \mu_K \)
- **Alternative hypothesis:** At least one of the means are different from the others.
- **F-test** (has an F-distribution with degrees of freedom \( K - 1, n - 1 \)):
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  F = \frac{SSG/(K - 1)}{SSW/(n - 1)}
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- Intuitively, what is implied when the F-statistic is large?
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Assumptions behind One-way ANOVA F-test

- **Randomness**: individual observations are assigned to groups randomly.
- **Independence**: individuals in each group are independent from individuals in another group.
- **Sufficiently large (?) sample size**, or else population must have a normal distribution.
- **Homogeneity of variance**: the variances of each of the $K$ groups must be equal ($\sigma_1^2 = \sigma_2^2 = \ldots \sigma_K^2$).
  - Levene test for homogeneity of variance can be used to test for this.
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Example: Crime Rates

- Data on 47 states from 1960 (I know it's old) on the crime rate and a number of factors that may influence the crime rate.
- In particular, I made a variable that put unemployment into categories:
  - Unemployment = 1 if unemployment rate was less than 8%.
  - Unemployment = 2 if unemployment rate was between 8 and 10%.
  - Unemployment = 3 if unemployment rate was greater than 10%.
- I also made a variable that categorized schooling:
  - Schooling = 1 if mean years of schooling for the given state was less than 10 years.
  - Schooling = 2 otherwise.
- Is there statistical evidence that the mean crime rate is different among the different categories for the level of unemployment?
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Kruskal-Wallis Rank Test: non-parametric technique for testing for differences in the *medians* among two or more groups.

- Like the Mann-Whitney U-test, uses information about the ranks of the observations, instead of the actual sizes.
- Null hypothesis: $\theta_1 = \theta_2 = ... = \theta_K$ (i.e. all groups have the same median).
- Alternative hypothesis: at least one of the medians differ.
- As the sample size gets large (over 5 per group some say!), the Kruskal-Wallis test statistic approaches a $\chi^2$ distribution with $K - 1$ degrees of freedom.
- For small sample sizes: possible to compute exact p-values without depending on asymptotic distributions.
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Assumptions for Kruskal-Wallis Test

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One-way ANOVA, the effects of one factor where examined.

Two-way ANOVA, also called two-factor factorial design: two factors are simultaneously evaluated.

Total variance is decomposed into:

- variability explained by being in different groups of factor A.
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### Goals of ANOVA:
- Determine if schooling level (factor A) leads to different levels for crime rate.
- Determine if unemployment level (factor B) leads to different levels for crime rate.
- Determine if schooling and unemployment have a joint effect on crime rate (i.e. does the unemployment level effect the impact of schooling on the crime rate, or vice versa).
### ANOVA Descriptive Statistics

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- \( H_0 : \mu_1 = \mu_2 = \ldots = \mu_r. \)
- \( H_a : \) At least one of the means of the groups in factor A are different from the others.

\[
F = \frac{SSA/(r-1)}{SSE/(N-rc)}
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- F-statistic has degrees of freedom \( r - 1, \) \( N - rc. \)
- \( N = \sum_{i=1}^{r} n_i. \)
- \( r \) (number of rows) is the number of groups for factor A.
- \( c \) (number of columns) is the number of groups for factor B.
- \( \mu_i. \) is the mean of group \( i \) of factor A.
- SSA is the sum of squares from factor A.
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F = \frac{SSA/(r - 1)}{SSE/(N - rc)}
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- F-statistic has degrees of freedom \( r - 1, \ N - rc \).
- \( N = \sum_{i=1}^{r} n_i \).
- \( r \) (number of rows) is the number of groups for factor A.
- \( c \) (number of columns) is the number of groups for factor B.
- \( \mu_i \) is the mean of group \( i \) of factor A.
- SSA is the sum of squares from factor A.
Hypothesis Test for Factor A

- \( H_0 : \mu_1 = \mu_2 = \ldots = \mu_r. \)
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Hypothesis Test for Factor B

- $H_0 : \mu_1 = \mu_2 = \ldots = \mu_c$
- $H_a : \text{At least one of the means of the groups in factor B are different from the others.}$

$$F = \frac{SSB/(c - 1)}{SSE/(N - rc)}$$

- F-statistic has degrees of freedom $c - 1, N - rc$.
- $\mu_j$ is the mean of group $j$ of factor B.
- SSB is the sum of squares from factor B.
Hypothesis Test for Factor B

- $H_0: \mu_1 = \mu_2 = \ldots = \mu_c$
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Hypothesis Test for Interaction of Factors A and B

- $H_0$: there is no interaction effect.
- $H_a$: there is an interaction effect.

$$F = \frac{SSAB/(r - 1)(c - 1)}{SSE/(N - rc)}$$

- F-statistic has degrees of freedom $(r - 1)(c - 1)$, $N - rc$.
- SSAB is the sum of squares from factors A and B.
Hypothesis Test for Interaction of Factors A and B

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Or.. run a Kruskal Wallis test with r x c different groups:

- Group 1: Unemployment=1, Schooling=1
- Group 2: Unemployment=2, Schooling=1
- Group 3: Unemployment=3, Schooling=1
- Group 4: Unemployment=1, Schooling=2
- Group 5: Unemployment=2, Schooling=2
- Group 6: Unemployment=3, Schooling=2

The Tukey pairwise tests for these combinations will indicate if there are interaction effects.
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